Modeling and optimization for prediction of moisture content, drying rates and moisture ratio

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Abstract: There were needs to develop some preservation technique to enhance the shelf life of Paneer because it is highly perishable in nature at ambient conditions. Drying can be one of the methods to increase shelf life of paneer. This study was undertaken to dry 1.5 cm^3 paneer at 62, 72 and 82℃ temperatures and 10, 14 and 18 kPa absolute pressures with superheated steam. Moisture content, drying rate and moisture ratio data were generated by conducting the experiments in low pressure superheated steam dryer. These data were used to develop Artificial Neural Network (ANN) models. Optimized ANN models were developed for rapid and more accurate prediction of moisture content with two hidden layers and seven nurons having R^2 0.9991, drying rate with two hidden layers and nine nurons having R^2 0.9846 and moisture ratio with two hidden layers and seven nurons having R^2 0.9991 in drying, based on two hidden layers and one to nine neurons in each hidden layer. Measured values of moisture content, drying rate and moisture ratio were predicted with an $R^2 > 0.98$. System equation has been developed to predict moisture content, drying rate and moisture ratio at any given conditions.

Key words: Artificial Neural Network(ANN), moisture content, drying rate, moisture ratio, optimization

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1 Introduction

Now days, superheated steam drying is interesting for food industries due to its several advantages. They are: higher porosity and hence better rehydration, absence of oxidative reactions (enzymatic browning, lipid oxidation) due to lack of oxygen, high heat transfer coefficients, higher drying rates, stripping of acids that contribute to an undesirable taste or aroma, energy saving due to recovery of latent heat supplied to the dryer, environment friendliness since it is a closed system, no explosion or fire hazard, combination of drying with other product treatments like pasteurization or sterilization of food stuffs[1,2]. The main objectives during the process control are high quality product with minimal cost and safety of food. To achieve these objectives, on-line control techniques are required. Food processes are highly nonlinear which complicates food process automation. However, recent developments in advanced control tools, such as artificial neural network (ANN) to food processing have opened up novel possibilities for processing industries[11]. A number of researchers have

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worked on ANN as a modeling tool in food technology. It has been successfully used in several food applications like model for prediction of drying rates, physical properties of dried carrot, prediction of dryer performance, extrusion processing of wheat and wheat-black soybean, energy requirements for size reduction of wheat, grain drying process, dough rheological properties among others[3-9]. This study has been undertaken to develop ANN model for prediction of moisture content (% dry basis), drying rates and moisture ratio for process optimization during low pressure superheated steam drying.

2 Materials and methods

Paneer with the brand name 'Anchal', prepared from standardized cow milk was procured from Anchal Dairy, Lalkuaon, Uttarakhand. It was kept at 4℃ in a refrigerator before being used. The initial moisture content of paneer was about 50% wet basis. It was diced into 1.5 cm^3 sizes with a stainless steel knife. Prior to drying, cubes were pretreated with sodium chloride and potassium sorbate. By this treatment paneer cubes could be successfully dried with no loss of fat and no browning[10]. After pretreatment, approximately 50 grams of sample were taken for drying. The drying chamber was sealed tightly. Before that, steam was generated till it reached 150 kPa gauge pressure which was maintained. A vacuum pump was switched on to evacuate the drying chamber to the desired operating pressure and then steam inlet valve opened slowly to flash steam into the drying chamber. Because of low pressure environment in the chamber, steam became superheated. At the end of drying, vacuum breakup valve was opened to allow air into the drying chamber followed by opening the chamber door and loading off the sample. The experiments were performed at 10, 14 and 18 kPa absolute pressures and 62, 72 and 82℃ steam temperatures respectively. During drying, the product temperature is equal to the wet bulb temperature, i.e. boiling temperature of water at given pressure. But in this study the main aim was to conduct the experiments at low pressures i.e. 10, 14 and 18 kPa with a corresponding temperature range of 60 to 85℃, which is boiling temperature of water.

A setup was developed in the laboratory with automatic data acquisition system. Experimental setup of low pressure superheated steam dryer with its accessories is shown schematically in Figure 1. The drying chamber consisted of a box insulated properly with rock wool. Inner dimensions of insulated chamber were 40 cm×45 cm×45 cm. Two electric heaters of 1.5 kW capacities each were provided on the opposite side walls of the drying chamber. The temperature of drying chamber was controlled by a temperature controller knob provided in front of drying chamber. Thermostat was provided to maintain the temperature inside the drying chamber. The drying chamber was connected by a pipe from bottom to a chamber in which digital balance was kept. An autoclave was used as a steam generator and a steam reservoir. Steam was transported to the drying chamber through a pipe insulated with glass wool. A heating tape, rated 1 kW was mounted on steam pipeline to increase the steam temperature to the desired level of superheating. Temperature was controlled by Dimmerstat. The sample holder was made using a thin stainless steel sheet into a circular disc with 15 cm diameter. This was connected to a balance by a thin rod passing through a G.I. pipe. One side of the rod was attached to the sample holder and other side was rested on analytical digital balance (model XB-320M, Adair Dutt & Co. (I) Pvt. Ltd., Kolkata). The balance was placed in the smaller chamber. It had a weighing capacity of 320 g with a least count of 0.001 g. The data recorded by this balance was transferred through the serial port by a software (Remote Control Version 1.0.38. Adair Dutt & Co. (I) Pvt. Ltd., Kolkata, India). Electronic balance attached to computer allowed continuous weighing of the sample. Chromel - Alumel (K type) thermocouples were installed to measure temperature of superheated steam at inlet of drying chamber, drying chamber, product and balance chamber continuously. These thermocouples were attached to the data logger (Model 1551C12, Digitech, Roorkee, India). It had eight-digit self-illuminating display. Thermocouple signals were multiplexed and transferred to the computer through Terminal Software, installed in PC. A vacuum pump (Kirloskar Pneumatic Co. Ltd, Poona, India) was used to create the desired vacuum in the drying chamber. All the experiments were carried out in triplicate.

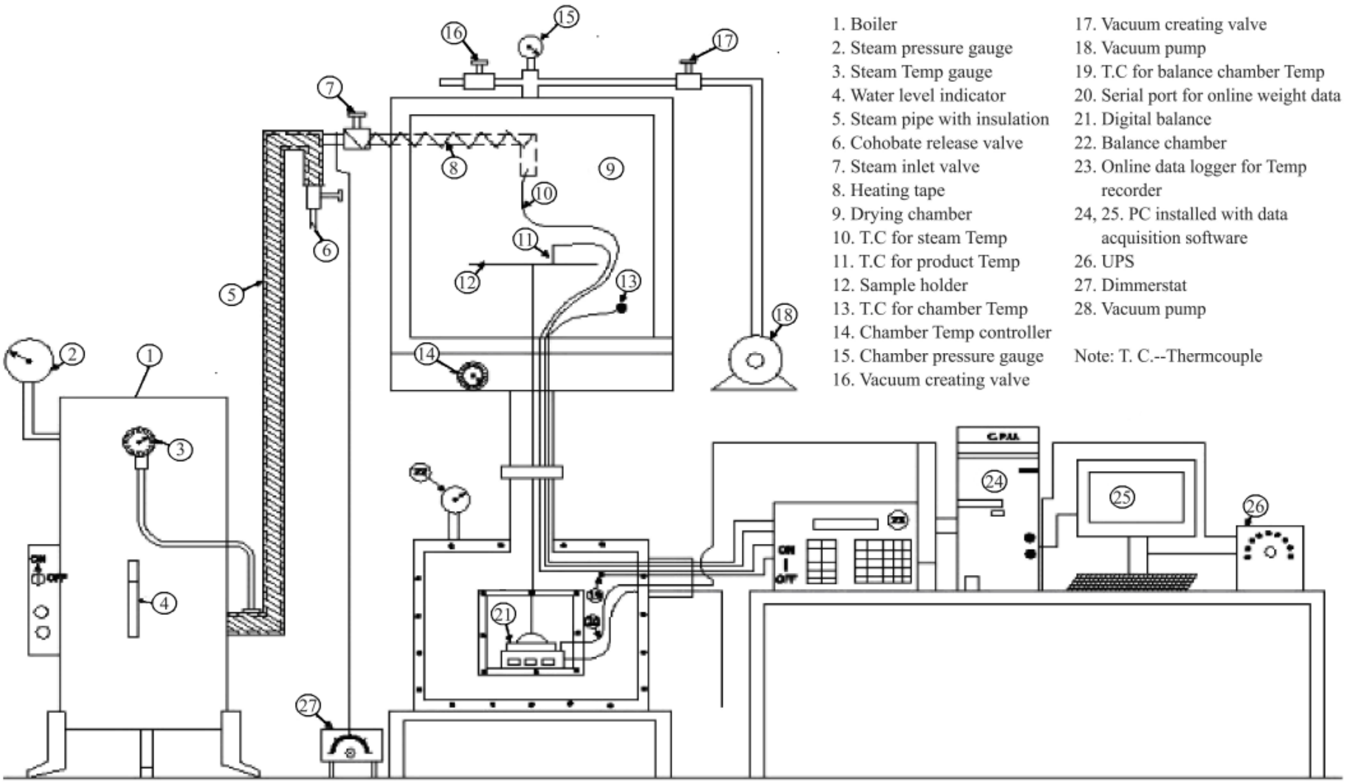

Figure 1 Schematic diagram of the low pressure superheated steam dryer and associated units

2.1 ANN description of the drying process

The neural network model consisted of an input, a hidden and an output layer was designed. The input layer has two nodes which correspond with processing conditions or independent variables: time of drying, corresponding weight of the sample, temperature, pressure and size of sample. The output layer consists of three neurons or dependent variables, representing the moisture content (%, d.b.), drying rate and moisture ratio. All three parameters were calculated with the following equations;

Moisture content

$$M = \frac{W_1 - W_2}{W_2} \times 100 \tag{1}$$

where M——Moisture content of sample, % (d.b.); W_1—— Weight of sample, g; W_2——Weight of bone dry sample, g.

Moisture Ratio (MR)

$$MR = \frac{M - M_e}{M_o - M_e} \tag{2}$$

where M —Average moisture content, (%,d.b.) at time t(min) during drying; M_o——Moisture content, (%, d.b.) at the initiation of drying i.e. at zero time; M_e—— Equilibrium moisture content, (%, d.b.).

Drying rate

$$\frac{dm}{dt} = \frac{M_2 - M_1}{\Delta t} \tag{3}$$

where Δt ——Difference in time.

The nodes and the neurons were connected to each other by weighted links, w_{ij}, over which signals can pass. The arriving signals multiplied by the connection weights are first summed (activation function) and then passed through the sigmoid function (transfer function) to produce the corresponding output that may be passed on to other neurons.

2.2 Training and testing algorithms

MATLAB-7 software was used for Artificial Neural Networks (ANN) modeling. The networks were simulated based on a multilayer feed forward neural network. This type of network is very powerful in function optimization modeling[7]. The input layer, hidden layers, and output layer structures are shown in Figure 2. The inputs required for modeling other than drying time and weights were:

- Method of computation -Back propagation
- Algorithm for minimization of error - Levenberg -Marquardt
- The network training - Different size of epochs
- Goal - Minimum error
- Transfer functions - Hyperbolic tangent, sigmoid transfer function and Linear transfer function

A back-propagation algorithm was used to implement supervised training of the network. During training, weighting functions for the inputs to each ANN were automatically adjusted such that the predicted output best matched with the actual output from the data set. Weights were randomly assigned at the beginning of the training phase according to the back-propagation algorithm. A hyperbolic tangent was selected as the transfer function in each hidden layer, and a linear transfer function for the output layer. Minimization of error was accomplished using the Levenberg – Marquardt (LM) algorithm. This algorithm trains a neural network 10 to 100 times faster than the usual gradient descent back propagation method. It will always compute the approximate Hessian matrix which had dimensions n-by-n. Training was finished when the mean square error (MSE) converged and was less than 0.001. If the MSE did not go below 0.001, training was completed after 5000 epochs, where an epoch represents one complete sweep through all the data in the training set.

The inputs included the time of drying, weight changed with time, temperature, pressure and size of sample. The output layer consisted of % (d.b.), dm/dt and MR. The number of hidden layers were two and the number of neurons in each hidden layer varied from 1 to 9 (3, 5, 7, or 9). The networks were simulated with the learning rate equal to 0.05. For training and testing of ANN configuration different ratio of data sets were examined. It was found that 50% of data set was used for training and 50% for testing predicted the best output.

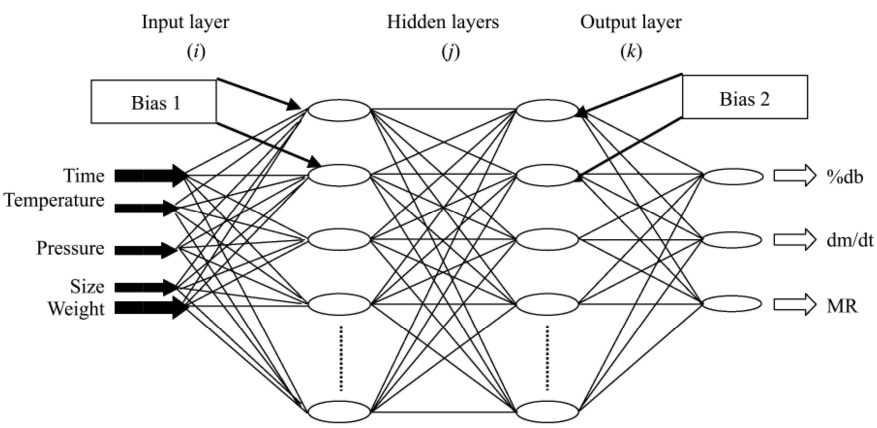

Figure 2 Theoretical architecture of multilayer neural network for prediction of moisture content, drying rate and moisture ratio

2.3 Optimization of ANN configuration

The optimal configurations from training and testing for each neuron were selected based on neural network predictive performance which gave the minimum error from training process. The mean relative error (*MAE*), Standard deviation of MAE (STD_A), Percentage of relative mean square error (%*MRE*), and standard deviation of %MRE (STD_R) were used to compare the performances of various ANN models and they were calculated as given below[7, 12].

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\Delta P_A \quad (4)$$

$$STD_A = \sqrt{\frac{\sum_{i=1}^{N}(\Delta P_A - \overline{\Delta P_A})^2}{N - 1}} \quad (5)$$

$$\%MRE = \left(\frac{1}{N}\sum_{i=1}^{N}\Delta P_R\right)\times 100 \quad (6)$$

$$STD_R = \sqrt{\frac{\sum_{i=1}^{N}(\Delta P_R - \overline{\Delta P_R})^2}{N - 1}} \quad (7)$$

where $\Delta P_A = \left| P_p - P_E \right|$, $\Delta P_R = \left| (P_p - P_E) / P_E \right|$.

3 Results and discussion

The ANN optimization process was performed using a trial and error technique. Time, weight, temperature, pressure and size of sample were used as input in the artificial neural network structure. The data set of inputs and outputs used to train the ANN consisted of combination of time of drying, corresponding weight, temperature, pressure and sample size of all superheated steam dried samples. Each data set was divided into two groups, consisting of 50% for training and 50% for testing. Total ninety data sets were divided into two groups because the network (ANN), which the authors have decided to develop and as per the selected data set. Once it is trained, it should be tested with the remaining data set. As far as 50% selection of data is concerned, author tested different combinations of data set like 70%～30%, 30%～70%, 60%～40% and 40%～60% and found that the best result was with 50%～50% data set. (The term testing and validation are the same, and some prefer to write testing and some validation.)

During training, the data set was used to determine the optimum number of hidden layers, neurons per hidden layer that gave the best predictive power. Architecture of artificial neural network was hidden layers 1 and 2 and neurons 3-9 per hidden layer. Each combination of hidden layers and neurons per hidden layer was trained. *MAE*, STD_A, % *MRE* and STD_R and R^2 along with number of hidden layers and neurons in each hidden layer are reported for one set of data for illustration in Table 1. In Table 1 the minimum MRE was found with two hidden layers and seven neurons for moisture content (dry basis) and moisture ratio where as for drying rate, minimum *MRE* was found at two hidden layers and nine neurons for any given set of data.

Table 1 Prediction of drying properties at low pressure superheated steam drying

No of hidden layer	No of neurons	(%)db					dm/dt					MR				
		MAE	***STDA***	***MRE***	***STDR***	R^2	***MAE***	***STDA***	***MRE***	***STDR***	R^2	***MAE***	***STDA***	***MRE***	***STDR***	R^2
1	3	2.883	2.820	9.550	0.112	0.9881	0.038	0.045	12.251	0.096	0.9575	0.020	0.021	6.603	0.078	0.9951
1	5	2.478	1.689	10.711	0.122	0.9987	0.038	0.044	12.147	0.088	0.9580	0.038	0.044	12.147	0.088	0.9580
1	7	3.813	2.989	14.014	0.167	0.9842	0.041	0.049	12.943	0.101	0.9554	0.042	0.033	15.036	0.188	0.9838
1	9	2.761	2.105	15.354	0.204	0.9983	0.040	0.048	12.711	0.098	0.9563	0.027	0.021	15.320	0.215	0.9983
2	3	3.779	3.034	13.797	0.164	0.9843	0.041	0.047	13.631	0.120	0.9664	0.040	0.033	14.591	0.184	0.9845
2	5	2.601	1.850	10.614	0.141	0.9933	0.039	0.046	11.816	0.082	0.9932	0.028	0.020	11.394	0.156	0.9932
2	7	1.002	0.947	3.333	0.043	0.9991	0.037	0.046	11.786	0.093	0.9634	0.011	0.010	4.208	0.059	0.9991
2	9	1.584	1.049	5.888	0.054	0.9980	0.034	0.043	10.574	0.093	0.9846	0.016	0.011	5.952	0.057	0.9980

The results showed that the number of hidden layers and neurons per hidden layer that yielded minimum error were not different for each drying technique. A large number of hidden layers were not required to lower the error if there were enough number of neurons[12]. The best prediction for most of the data set contained two hidden layers. ANN developed for combined drying data had slightly higher error than that under individual conditions. For all combined data set with superheated steam, the R^2 was found greater than 0.98 for moisture content, drying rate and moisture ratio, respectively. This shows that the ability of ANN to predict moisture content, drying rate and moisture ratio was very good. The system equations representing the ANN for predicting moisture content, drying rate and moisture ratio are given in appendix 1. The system equations show the input, transfer function, and relative weights of each node. The equations can be used in computer program to predict the moisture content, drying rate and moisture ratio of paneer cubes for any given set of conditions.

4 Conclusion

ANN models were developed with a wide range of data obtained by combining all data sets. Artificial neural networks can be used to predict the moisture content, drying rate and moisture ratio of products for different drying techniques For all combination data set in low pressure superheated steam drying, the correlation coefficient (R^2) was found greater than 0.98, offering better predictive performance than that with traditional regression techniques. Also, system equation can give direct prediction of moisture content, drying rate and moisture ratio by putting different input variables into it. Thus, ANN modeling can be successfully used for accurately predicting moisture content, drying rate and moisture ratio.

Nomenclature

MR	Moisture ratio
P_p	Predicted out put (%, db, dm/dt, MR)
P_E	Experimentally measured output
X	Moisture content
W	Weight of sample
t	Time of drying
T	Temperature of drying
P	Pressure
S	Size of sample
(dm/dt)	Drying rate

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Appendix

System Equations

For the prediction of moisture content (%db)

$X_1 = \tanh[(0.0097)*X+(0.3181)*W+(-0.5282)*t+(0.202)*T+(-0.6861)*P+(0.4902)*S+(-17.2556)]$

$X_2 = \tanh[(0.0101)*X+(0.3166)*W+(0.2015)*t+(0.3701)*T+(0.2805)*P+(-0.1831)*S+(-16.0134)]$

$X_3 = \tanh[(0.6425)*X+(-0.1811)*W+(0.2002)*t+(0.4654)*T+(-0.1542)*P+(0.2053)*S+(1.0550)]$

$X_4 = \tanh[(0.0034)*X+(0.3344)*W+(-0.0354)*t+(-0.7985)*T+(0.4779)*P+(0.0701)*S+(-13.3302)]$

$X_5 = \tanh[(0.0149)*X+(-0.2915)*W+(0.0136)*t+(-1.7441)*T+(0.2478)*P+(0.0229)*S+(10.4455)]$

$X_6 = \tanh[(0.0018)*X+(-0.3360)*W+(-1.0242)*t+(0.4003)*T+(0.6847)*P+(-0.5026)*S+(15.1535)]$

$X_7 = \tanh[(0.0100)*X+(0.3176)*W+(0.8187)*t+(-0.7161)*T+(0.2052)*P+(-1.0803)*S+(-9.8466)]$

X_8=tan$h$[(0.363)*X_1+(-0.7813)*X_2+(0.1559)*X_3+(-0.0964)*X_4+(0.9973)*X_5+(-0.4303)*X_6+(-1.2146)*X_7 +(-1.8497)]

X_9= tan$h$[(-0.0494)*X_1+(-1.1341)*X_2+(1.022)*X_3+(0.2635)*X_4+(0.4082)*X_5+(1.0303)*X_6+(-1.1005)*X_7 +(1.0534)]

X_{10}=tan$h$[(-1.0779)*X_1+(-1.0832)*X_2+(-1.1977)*X_3+(0.875)*X_4+(-0.7163)*X_5+(0.842)*X_6+(0.2885)*X_7 +(-0.6243)]

X_{11}=tan$h$[(-1.0437)*X_1+(-0.9508)*X_2+(0.6363)*X_3+(1.4306)*X_4+(0.6559)*X_5+(-0.1368)*X_6+(-0.4769)*X_7 +(-0.057)]

X_{12}=tan$h$[(0.0077)*X_1+(-0.0515)*X_2+(0.6334)*X_3+(0.8875)*X_4+(-0.0921)*X_5+(0.6816)*X_6+(-0.0716)*X_7 +(0.5633)]

X_{13}=tan$h$[(-0.6421)*X_1+(-0.0218)*X_2+(0.3916)*X_3+(0.5396)*X_4+(0.9791)*X_5+(0.5861)*X_6+(0.6669)*X_7 +(-1.4508)]

X_{14}=tan$h$[(0.7909)*X_1+(-1.1484)*X_2+(0.0175)*X_3+(0.4925)*X_4+(0.7353)*X_5+(0.0654)*X_6+(0.0588)*X_7 +(-1.8399)]

X_{15}=purlin[(0.0159)*X_8+(-1.4025)*X_9+(0.127)*X_{10}+(-1.2152)*X_{11}+(0.0107)*X_{12}+(-0.8357)*X_{13}+(-0.1092)*X_{14} +(-1.1088)]

%d.b. = ((0.848)* X_{15}+ (0.415))

For the prediction of drying rate

X_1 = tan$h$[(-0.0233)*X+(-0.1041)*W+(-0.6129)*t+(0.7439)*T+(-0.0086)*P+(-0.2301)*S+(10.2816)]

X_2 = tan$h$[(-0.3271)*X+(1.2529)*W+(-0.5634)*t+(-0.3606)*T+(-0.1749)*P+(-0.0186)*S+(-0.0695)]

X_3= tan$h$[(0.0124)*X+(-0.2476)*W+(0.1857)*t+(0.2748)*T+(0.399)*P+(-0.2617)*S+(7.862)]

X_4= tan$h$[(-0.0182)*X+(-0.1961)*W+(0.0261)*t+(0.0132)*T+(0.379)*P+(0.0189)*S+(8.4308)]

X_5= tan$h$[(0.005)*X+(-0.2789)*W+(0.6699)*t+(0.7932)*T+(0.6305)*P+(0.5885)*S+(13.1905)]

X_6= tan$h$[(0.8936)*X_1+(1.6536)*X_2+(-0.5165)*X_3+(0.3501)*X_4+(-0.3892)*X_5+(-1.9825)]

X_7= tan$h$[(-0.9668)*X_1+(-0.7822)*X_2+(-0.947)*X_3+(-0.7253)*X_4+(-0.7199)*X_5+(1.0136)]

X_8= tan$h$[(1.2624)*X_1+(-0.989)*X_2+(1.0186)*X_3+(-0.3806)*X_4+(-0.5801)*X_5+(0.1690)]

X_9= tan$h$[(-0.513)*X_1+(-0.7714)*X_2+(1.0709)*X_3+(-1.0464)*X_4+(0.8953)*X_5+(-1.1177)]

X_{10}= tan$h$[(0.7447)*X_1+(-1.6032)*X_2+(-0.9085)*X_3+(-0.603)*X_4+(-0.1497)*X_5+(2.0761)]

X_{11}= purlin[(1.027)*X_6+(-0.5584)*X_7+(-1.2743)*X_8+()-1.0635*X_9+(0.3642)*X_{10}+(0.4652)]

dm/dt = ((0.348)* X_{11}+(0.145))

For the prediction of moisture ratio

X_1 = tan$h$[(-0.4221)*X+(0.1026)*W+(0.7239)*t+(-0.0284)*T+(0.6441)*P+(-1.0071)*S+(2.5147)]

X_2 = tan$h$[(-0.0187)*X+(0.2601)*W+(-0.3103)*t+(-0.4506)*T+(0.0996)*P+(0.1039)*S+(-4.9969)]

X_3 = tan$h$[(0.1892)*X+(-0.2659)*W+(0.7149)*t+(0.1669)*T+(0.2831)*P+(-0.9278)*S+(0.3506)]

X_4 = tan$h$[(0.0227)*X+(0.2159)*W+(0.3634)*t+(-0.9458)*T+(0.1031)*P+(0.983)*S+(-11.3791)]

X_5 = tan$h$[(0.017)*X+(0.2755)*W+(0.0264)*t+(-0.0218)*T+(0.656)*P+(0.1519)*S+(-11.6675)]

X_6 = tan$h$[(0.0264)*X+(0.152)*W+(-0.0181)*t+(0.3817)*T+(0.0756)*P+(0.2848)*S+(-6.9478)]

X_7 = tan$h$[(-0.0347)*X+(-0.0958)*W+(0.0579)*t+(-1.0554)*T+(0.2934)*P+(0.3367)*S+(3.9944)]

X_8=tan$h$[(-0.8162)*X_1+(-0.1338)*X_2+(-1.2485)*X_3+(0.2857)*X_4+(1.0379)*X_5+(0.1429)*X_6+(-0.2421)*X_7 +(1.8824)]

X_9= tan$h$[(-0.9602)*X_1+(-0.214)*X_2+(0.8344)*X_3+(0.6313)*X_4+(0.9722)*X_5+(0.3203)*X_6+(0.4732)*X_7 +(1.3156)]

X_{10}=tan$h$[(-1.1018)*X_1+(-0.1888)*X_2+(0.5199)*X_3+(-0.4583)*X_4+(-0.4876)*X_5+(-0.9678)*X_6+(0.7283)*X_7 +(0.6275)]

X_{11}=tan$h$[(0.0495)*X_1+(-0.4792)*X_2+(0.8272)*X_3+(-1.5158)*X_4+(0.0583)*X_5+(-0.1273)*X_6+(-0.012)*X_7 +(0.2529)]

X_{12}=tan$h$[(0.8369)*X_1+(1.0517)*X_2+(-0.1213)*X_3+(-0.1555)*X_4+(0.6248)*X_5+(1.1266)*X_6+(-0.262)*X_7 +(0.5709)]

X_{13}=tan$h$[(-0.0568)*X_1+(-1.3154)*X_2+(-0.3504)*X_3+(0.7151)*X_4+(-0.8043)*X_5+(-0.4208)*X_6+(0.3947)*X_7 +(-1.259)]

X_{14}= tan$h$[(0.5064)*X_1+(-0.2462)*X_2+(-1.4408)*X_3+(0.919)*X_4+(1.1795)*X_5+(0.5942)*X_6+(0.563)*X_7 +(1.6196)]

X_{15}=purlin[(0.6004)*X_8+(-0.6643)*X_9+(0.1283)*X_{10}+(0.2583)*X_{11}+(0.3765)*X_{12}+(0.2854)*X_{13}+(1.3626)*X_{14} +(-0.0301)]

MR=((0.738)*X_{15} + (0.56215))