Analysis and finite element simulation for mechanical response of wheat to wind and rain loads

Liang Li, Guo Yuming

(*College of Engineering and Technology*, *Shanxi Agricultural University*, *Taigu* 030801, *China*)

Abstract: The morphological characteristics and stalk biomechanical properties at the dough stage of wheat were determined using the variety of wheat in the breeding process. Their mechanical responses to wind and rain loads for an individual and a group of wheat were simulated using finite element method by ANSYS. The stress and displacement of each finite element can be outputted through stress nephogram and displacement nephogram, respectively. In order to judge whether the wheat could return to its original position after deformation, elastic mechanics theory was utilized to analyze the critical load of instability under both axial rain load and transverse wind load. The large displacement situation was analyzed with large displacement elastic nonlinear theory and the numerical value was obtained by ANSYS. The results show that it is possible to apply various load types on models using ANSYS and the dynamic response can be simulated well under different rain and wind loads. The location of maximum Von Mises stress can be calculated and the variation of stress can be described clearly, which are helpful to predict the wheat lodging under wind and rain loads.

Keywords: wheat, wind and rain loads, finite element analysis, mechanical response

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1 Introduction

Wheat lodging is one of the most important factors affecting wheat yield, and the lodging resistance of wheat is regarded as an important index for breeding. Many researchers have studied the wheat lodging resistance. For example, establishment of mechanical model of crops is a popular method to evaluate the wheat lodging resistance through analyzing the mechanical response of wheat stalk to wind. Some theoretical models[1-4] were introduced to analyze the natural frequencies of cereals and trees under wind. Berry, et al. described a calibrated model[6] based on the existing models[5] and these models could be used to predict the timing and amount of lodging. However, most of those models require pure theory analysis and their calculation processes are extremely complicated. In some cases, the numerical solution can only be obtained by mathematical softwares[7]. On the one hand, some tests for determining the breaking resistance of crop stalk were performed to predict the lodging resistance of crops[8-10]. For instance, Berry P M[10] introduced a test instrument to measure the resistance of shoots against rotational displacement. He concluded that both stem and root lodging could be assessed. There are some other studies focusing on determining the morphological and mechanical properties of crop stalks to predict the lodging resistance[11-13]. However, the methods to obtain these mechanical indices are usually at static load, which are different from the dynamic process under wind and rain. On the other hand, field tests of wheat were carried out in a portable wind tunnel[14], which could simulate the mechanical response of wheat close to the natural situation, but it is difficult to design, test and control the system of wind tunnel very costly.

With the development and application of computer technology in agriculture, the mechanical response of wheat to wind and rain loads can be simulated by computer simulation softwares recently. It cannot only avoid the design of test apparatus and the complicated calculation, but also can express the mechanical response and stress changing status clearly. Therefore, aiming at studying the mechanical response of wheat to wind and rain loads, elastic mechanics theory was utilized in this study to analyze the critical load of instability under both axial rain load and transverse wind load. The large displacement situation was also analyzed with large displacement elastic nonlinear theory. A finite element

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Author: Liang Li (1982－), born in Linfen City, Shanxi Province, Ph.D. student at College of Engineering and Technology, Shanxi Agricultural University. Taigu, 030801. Major in Agricultural biomechanics and material mechanical characteristics. Email: liangli_alice@163.com

Corresponding author: Guo Yuming (1954－), professor, doctor tutor，Dean.. Major in Agricultural biomechanics and material mechanical characteristics. College of Engineering and Technology, Shanxi Agricultural University, Taigu, 030801, China. Email: guoyuming99@sina.com

model of wheat under wind and rain loads was developed by ANSYS to simulate the dynamic responses of wheat and the stress and displacement nephograms were outputted.

2 Analysis of critical load of instability for wheat stalk under wind and rain loads

The diameter of wheat stalk is basically decreased gradually from the bottom to the top except the first basal internode, so that it can be simplified as a slender stalk with varying rigidity. Because some basic parameters need to be input for establishment of the finite element model, the wheat "Shannong 129" was chosen to determine the morphological and biomechanical properties. The mean values of the properties are listed in Table 1.

Table 1 Mean values of wheat morphological traits and biomechanical properties

Internode number	Internode distance /mm	Outside diameter /mm	Wall thickness /mm	Ear mass /g	Young's Modulus /MPa	Shear elastic modulus /MPa	Yield stress /MPa	Bending rigidity /N.mm^2	Poisson's ratio
1	78.49	3.63	0.83		2168.62	757.73	20.11	16203.73	
2	118.69	3.60	0.54		2228.98	778.82	15.07	14601.44	
3	139.80	3.68	0.45	3.89	1429.63	499.52	10.49	8925.47	0.413
4	176.76	3.58	0.37		1458.55	509.63	9.92	7256.45	
5	284.86	2.96	0.43		1973.65	689.61	18.5	5464.96	

The mechanical analysis of wheat in this paper is mainly under rain and wind loads, and the force diagram of wheat is shown in Figure 1a.

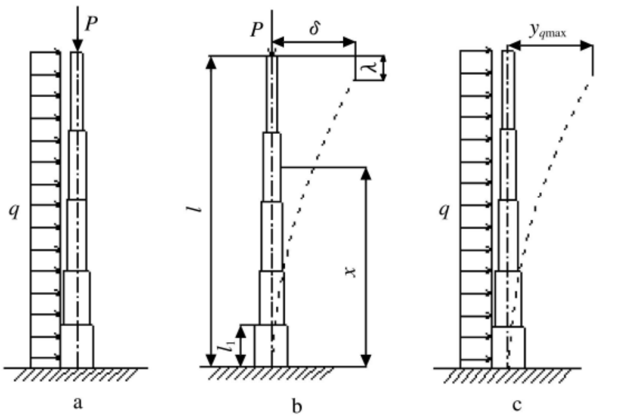


Figure 1 Force diagram of member with varying rigidity

2.1 Solving the Eular critical load of varying rigidity member under axial load P

According to Energy Method, the deflection curve of compression member shown in Figure 1b can be supposed as

$$v = \delta\left[1-\cos\left(\frac{\pi x}{2l}\right)\right] \quad (1)$$

Where, x is the distance from stalk base, l is the whole length of wheat stalk and δ is the horizontal displacement at the top of stalk. Because the above equation satisfied the boundary conditions, the bending moment of random cross-section is

$$M(x) = P(\delta - v) = P\cos\left(\frac{\pi x}{2l}\right) \quad (2)$$

Thus, the energy increment of bending deformation is

$$\Delta U = \sum_{i=0}^{4}\int_{l_i}^{l_{i+1}}\frac{M^2}{2E_iI_i}dx = \frac{P^2\delta^2}{4}\sum_{i=0}^{4}\frac{\left[x+\frac{l}{\pi}\sin\left(\frac{\pi x}{l}\right)\right]_{l_i}^{l_{i+1}}}{E_iI_i} \quad (3)$$

Where l_i (i=0,1,2,3,4) is the coordinate about top of internode along the member height, and E_iI_i is the bending rigidity of each internode.

In addition, the work done by compression (P) during member deforming is

$$\Delta W = P\lambda = \frac{\pi^2P\delta^2}{16l} \quad (4)$$

Where, λ is the vertical displacement during member bending.

Since $\Delta U = \Delta W$, the critical compression under axial load can be obtained as

$$P_{cr} = \frac{\pi^2}{4l\sum_{i=0}^{4}\frac{\left[x+\frac{l}{\pi}\sin\left(\frac{\pi x}{l}\right)\right]_{l_i}^{l_{i+1}}}{E_iI_i}}, \quad (5)$$

As an example, putting the tested data into the above equation, the Eular critical compression of wheat variety named "Shannong 129" is P_{cr} =0.0385*N*.

2.2 Calculation of the transverse critical load q_{cr} under both axial rain load and transverse wind load

At the random cross-section x, the bending moment is

$$M(x) = \frac{1}{2}qx^2 + Py \quad (6)$$

Where, y is the transverse displacement under both rain load and wind load. It is nonlinear with respect to P, but linear with q. It can be considered elastic within the critical state. Deducing the bending moment and deflection equation to calculate the maximum bending moment and maximum deflection, the transverse critical load becomes the load when maximum compression stress of member reaches yield strength. Therefore, it is analyzed as follows[15]: Assuming that the deflection caused by q is y_q, and the deflection caused by P is y_P, the deflection at random cross-section is $y = y_q + y_P$. The corresponding bending moment is $M=M_q+P_y$. Besides, it is assumed that $y_P=\beta y_q$, where β is non-dimensional coefficient, so that $y = \frac{1+\beta}{\beta} y_P$.

Therefore, the differential equation of deflection curve under axial compression is

$$EIy_P'' = -P\frac{1+\beta}{\beta} y_P \tag{7}$$

Let $k^2 = \frac{P(1+\beta)}{EI\beta}$, then $y_P'' + k^2 y_P = 0$

The general solution is

$$y_P = A\sin kx + B\cos kx \tag{8}$$

Considering the boundary conditions, then

$$\beta = \frac{P}{P_{cr} - P} \tag{9}$$

The maximum bending moment is calculated by

$$M_{\max} = \frac{ql^2}{2} + P_{cr} y_{\max} \tag{10}$$

Where, $y_{\max} = y_{q\max} + y_P = (1+\beta) y_{q\max} = \frac{ql^4}{8EI}(1+\beta)$,

where $y_{q\max}$ is the maximum deflection under q, as shown in Figure 1c.

As long as the maximum compression stress is $\sigma_{\max y} = \frac{M_{\max}}{W_z} + \frac{P}{A} = \sigma_s$, the transverse critical load is as follows[16].

$$q_{cr} = \frac{2(\sigma_s - \frac{P}{A})W_z}{l^2\left[1 + P_{cr}(1+\beta)\frac{l^2}{4EI}\right]} \tag{11}$$

Because lodging probably occurs at the basal second internode, when taking the tested data of basal second internode as an example to calculate the transverse critical load, the solution is $q_{cr} = 3.722\times10^{-5}$ N/mm.

2.3 Nonlinear large deformation analysis of wheat stalk

When member deformation exceeds critical position, it becomes a problem of large deformation. Hence the nonlinear large deformation theory can be used for the analysis.

The large deformation constitutive equation reflects the relationship between Kitchhoff stress tensor and Green strain tensor. Kitchhoff stress tensor is expressed as follows [17].

$$s_{ij} = J\frac{\partial X_i}{\partial x_k}\frac{\partial X_j}{\partial x_l}\sigma_{kl} \tag{12}$$

Where, J is Wronski determinant, $\frac{\partial X_i}{\partial x_k}$ is the increment of deformation gradient, σ_{ij} is the stress tensor, which is derived by the coordinate transformation of Euler stress tensor (σ_{ij}), and s_{ij} is symmetrical tensor, which changes with general rigid motion at the space coordinate.

Strain is expressed as Green strain tensor

$$E_{ij} = \frac{1}{2}\left(\frac{\partial x_k}{\partial X_i}\frac{\partial x_k}{\partial X_j} - \delta_{ki}\right) \tag{13}$$

Where, δ_{ij} is the Kronecker symbol. According to Stress tensor equilibrium equation and boundary conditions, the initial configuration can be illustrated as:

$$\begin{cases} [(\delta_{ik} + u_{i,k})s_{jk}]_{,j} + F_i = 0 & X \in v \\ (\delta_{ik} + u_{i,k})s_{jk}n_j = \overline{p_i} & X \in S_P \\ u_i = \overline{u_i} & X \in S_u \end{cases} \tag{14}$$

Formula (14) shows that the equilibrium equation of nonlinear elastic mechanics is coupled by stress tensor and displacement component since there is displacement component u_i in it. That is much more complicated than linear elastic equilibrium equation.

General constitutive equation is usually expressed by strain energy density function.

$$s_{ij} = \frac{\partial W(E_{ij})}{\partial E_{ij}} \tag{15}$$

Because $c_{ij} = \delta_{ij} + 2E_{ij} = \frac{\partial x_k}{\partial X_j}\frac{\partial x_k}{\partial X_j}$,

where c_{ij} is right Cauchy-Green strain tensor.

Hence, $s_{ij} = \frac{\partial W}{\partial c_{mn}}\cdot\frac{\partial c_{mn}}{\partial E_{ij}} = 2\frac{\partial W}{\partial c_{ij}}$.

If the material is supposed to be isotropic, the strain energy density function in the orthogonal transformation is constant. Moreover, the three strain tensor invariants of symmetrical tensor C_{ij} are

$$I_1 = c_{kk}, I_2 = \frac{1}{2} e_{ijk} e_{ilm} c_{jl} c_{km}, I_3 = \frac{1}{6} e_{ijk} e_{lmn} c_{il} c_{jm} c_{kn} \quad (16)$$

Therefore,

$$W = W(c_{ij}) = W(I_1, I_2, I_3) \quad (17)$$

$$s_{ij} = 2\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial c_{ij}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial c_{ij}} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial c_{ij}}\right) \quad (18)$$

Due to $\sigma_{ij} = J^{-1}\frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_l} s_{kl}$,

Then,

$$\sigma_{ij} = \frac{2}{\sqrt{I_3}}\frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_l}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial c_{kl}} + \frac{\partial w}{\partial I_2}\frac{\partial I_2}{\partial c_{kl}} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial c_{kl}}\right)$$

This shows the relationship between Eular stress tensor and Cauchy-Green strain tensor.

Based on Green Method, the constitutive equation, which is expressed by strain energy density function W, is constructed as follows.

$$\sigma_{ij} = \frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_l}\left[2\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial c_{kl}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial c_{kl}}\right) - P\frac{\partial X_k}{\partial x_m}\frac{\partial X_l}{\partial x_m}\right]$$

$$= 2\frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_l}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial c_{kl}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial c_{kl}}\right) - P\delta_{ij}$$

It can be simplified as follows.

$$\sigma_{ij} = \frac{2}{\sqrt{I_3}}\left[\left(\frac{\partial W}{\partial I_1} + I_1\frac{\partial W}{\partial I_2}\right)B_{ij} - \frac{\partial W}{\partial I_2}B_{ik}B_{kj} + I_3\frac{\partial W}{\partial I_3}\delta_{ij}\right] \quad (19)$$

or

$$\sigma_{ij} = 2\left[\left(\frac{\partial W}{\partial I_1} + I_1\frac{\partial W}{\partial I_2}\right)B_{ij} - \frac{\partial W}{\partial I_2}B_{ik}B_{kj}\right] - P\delta_{ij} \quad (20)$$

where, $B_{ij} = \frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_k}$ is left Cauchy-Green strain tensor and P is proportional constant, which is the function of coordinate.

The equation's boundary value problem is extremely complicated, as a result, numerical solutions are usually obtained by finite element method.

3 Results and analysis of finite element simulation

3.1 Finite element model of wheat

In the process of establishment of wheat finite element model, Beam188 (defined in ANSYS) shown in Figure 2 is chosen to be the element type. It is suitable for analyzing slender beam structure. It is based on Timoshenko beam theory and also considered shear deformation effects. The cross-section characteristics of beam can be shown visibly.

Biomaterials including wheat stalk have more complicated properties than those of engineering materials. It strictly belongs to visco-elasto-plastic material. However, it is difficult to test the related index, so the stalk material in this work is assumed as bilinear isotropic hardening material (shown in Figure 3). The parameters of material properties, such as elastic modulus, Poisson's ratio, yield stress and shear modulus are defined according to Table 1 from the bottom to the top of the stalk.

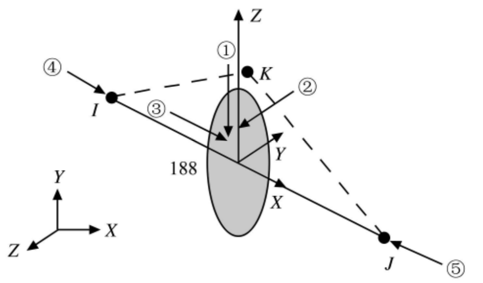


Figure 2 Beam 188 geometry

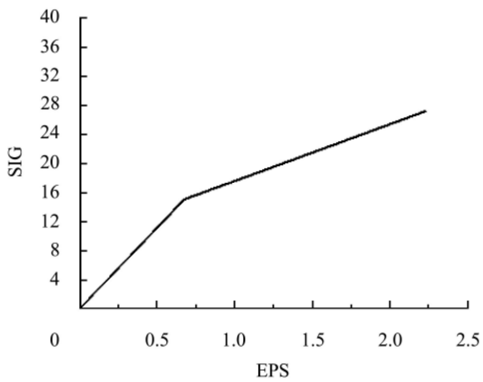


Figure 3 Material model of basal second stalk

Wheat stalk is simplified as cantilever beam of varying rigidity with an apical load[6]. It is negligible mass of the stalk. The beam is defined as a thin-walled hollow cylinder. The load-applied model is shown in Figure 1a. Ear weight and rain load are the axial loads which are applied to the stalk as concentrated force. Wind load is applied as a pressure on the beam which is calculated by the following formula.

$$\omega = \frac{\upsilon^2}{1600} \quad (21)$$

Where, υ is wind speed.

3.2 Static simulation of an individual wheat stalk

Various displacements and stresses for a single stalk under different loads were obtained by changing load type and size. The results are presented as displacement and stress nephograms. Take the wind speed of 3.4m/s as an example, the maximum displacement down the wind direction is 353.634 mm, the vertical maximum displacement is 119.467 mm and the maximum Von Mises stress is 4.52 MPa, which occurs at the basal second internode. At wind speed of 14.08m/s, the stalk reaches closely its yield stress. Von Mises stress nephograms for single wheat stalk under the wind speeds above are shown in Figure 4.

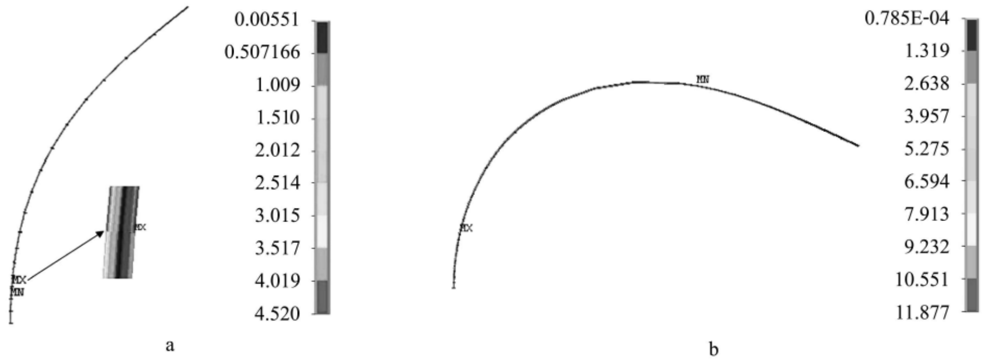


Figure 4 Von Mises stress (/MPa) nephograms of finite element at the wind speeds of 3.4 m/s (a) and 14.08 m/s (b)

When stalk exceeds its yield stress, it begins to deform plastically. At that time the strain is unrecoverable. Hence, lodging might occur if wind continues. The displacement and stress responses are shown in Figure 5.

If both rain and wind loads are considered together, when ear weight adds rain load of 0.01 N, according to the test, the maximum Von Mises stress is 7.945 MPa at wind speed of 3.4 m/s (Figure 6). The maximum displacement in horizontal direction is 525.412 mm, and in vertical direction is 331.093 mm. The maximum Von Mises stress of stalk increases by 75.8% compared with the situation when wind load acts alone.

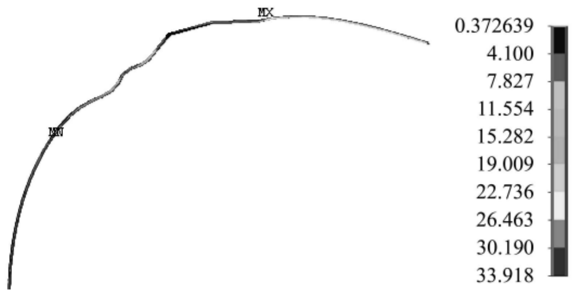


Figure 5 Stalk failure when the wind speed exceeds 14.08 m/s

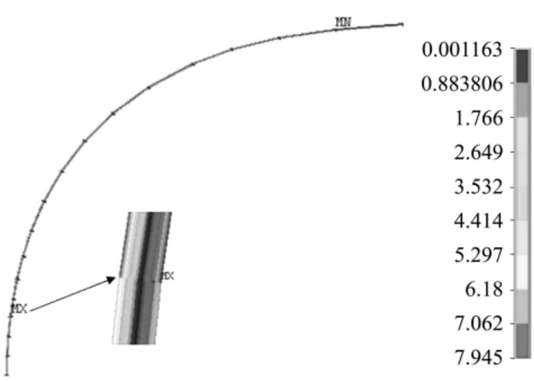


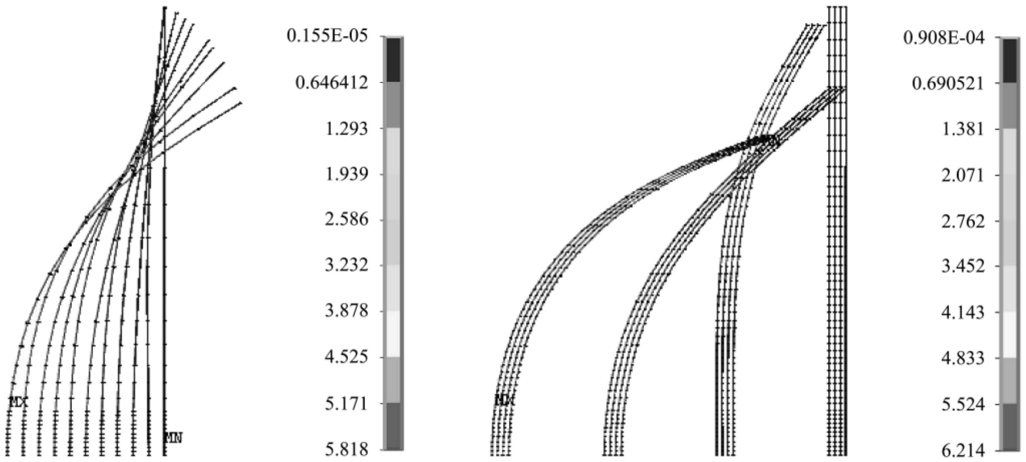
Figure 6 Stalk Von Mises stress nephogram considered rain load (0.01 N) at the wind speed of 3.4 m/s

3.3 Results of transient dynamic analysis about groups of wheat stalks

Transient dynamic analysis is used to determine the dynamic response of stalk structure under load varying with time. Based on the static analysis of mechanical response for an individual stalk, the transient dynamic analysis for a group of wheat (row planting) is simulated. The aim is to study the group of wheat stalk mechanical response and their interaction under wind load.

For the situation of wind blows parallel to row wheat, it is assumed that the initial displacement and speed are zero. According to load steps, loads are applied accordingly, which vary with time. When the first wheat stalk touches the second after wind load acting certain time, the second one begins to motion with the first one together as one system. The wind load decreases to half of the initial value, while the top of the second stalk needs a concentrated load caused by wind load acting on ear. When the second stalk touches the third one, the first three stalks start to motion together. The wind load decreases to 1/3 of the initial value, and it is the third one’s turn to add concentrated force at the top while the others become zero, because their ears are nearly horizontal. In a similar way, the transient dynamic response of many stalks can be simulated, and the displacement and stress of them under dynamic loads can be observed. Figure 7a shows the dynamic simulation results of ten stalks at the wind speed of 3.4 m/s.

For the situation of wind blows vertical to row wheat, the same analysis method as the above was used for the transient dynamic analysis, but the initial wind load was supported by each stalk in the first row. For example, it is supposed that there are four groups in a row of stalks. Then, the load applied on each group is 1/4 of the wind pressure. The next load step is to apply pressure with 1/8 of wind pressure on each stalk of the two rows with a concentrated force on the second row. In a similar way, four rows' dynamic responses were simulated by transient analysis method. The displacement and Von Mises stress of the first three rows are shown in Figure 7b.



a. Ten stalks in a row at wind speed of 3.4 m/s b. Four rows of stalks at the wind speed of 11.3 m/s

Figure 7 Von Mises stress of wheat stalks by transient dynamic analysis

4 Conclusions

1) From the static simulation of an individual wheat stalk, it can be concluded that the tested wheat stalk reaches its maximum elastic deformation when the wind speed is 14.08m/s. The wheat stalk will deform plastically when the wind speed continuously increases.

2) The varying process for groups of wheat stalk mechanical responses was observed through the transient dynamic analysis. The position where the maximum Von Mises stress occurred was obtained.

3) The critical loads of varying rigidity member under both rain load and wind load were calculated to judge when instability and lodging would occur. The large deformation state was analyzed by nonlinear large deformation theory when displacement exceeded the critical state. The stress equation can be used for analyzing the lodging mechanism.

4) The finite element model and analysis method in the present work have potential application to other crop stalks.

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