

Measurement of stiffness and damping coefficient of rubber tractor tires using dynamic cleat test based on point contact model

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Abstract: The ride vibration of a tractor is affected mostly by the stiffness and damping coefficient of the seat suspension, cabin suspension, cabin rubber mounts, and rubber tires. However, in the case of rubber tractor tires, the stiffnesses and damping coefficients have not been researched adequately thus far, and it is not simple to measure these characteristics. In this study, a method for measuring and analyzing the stiffnesses and damping coefficients of rubber tractor tires, which were the input parameters for the tractor ride vibration simulation, was proposed. The cleat test, proposed in this study, did not require separate and complicated test equipment, unlike the conventional methods. The test was conducted simply by measuring acceleration under the driving conditions of the vehicle without detaching tires from the vehicle body or setting up additional test equipment. Based on the ground-vertical acceleration data obtained, the stiffness was calculated using the logarithmic decrement method, and the damping coefficient was calculated using least squares exponential curve fitting. The result of the cleat test indicated that the front tires had stiffnesses of 486.08-570.69 kN/m and damping coefficients of 4.02-4.52 kN·s/m; the rear tires had stiffnesses of 409.42-483.79 kN/m and damping coefficients of 2.21-2.67 kN·s/m. During the test, 40 mm height cleats were installed on the track and the speed of the tractor was set to 7 and 10 km/h, which were the most common speeds during the operation. This study is meaningful in that it has presented a new method that improves the practicality of results, reduces cost, and simplifies the test process for measuring the stiffnesses and damping coefficients of rubber tractor tires.

Keywords: ride vibration, rubber tractor tire, stiffness, damping coefficient, cleat test, point contact model

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1 Introduction

As the ergonomics design of vehicles is highlighted, the negative impacts of ride vibration on the human body and ways to reduce them are being researched in various ways. Normally, the vibration characteristics of an object can be parameterized with its mass (m), stiffness (k) and damping coefficient (c). Therefore, these factors are essential when modelling a vehicle system to predict the vehicle's vibration characteristics or to design a vehicle through simulation.

Unlike regular passenger cars, agricultural vehicles are produced in small quantities but various models. Consequently, conducting performance testing for every single development model is difficult and costly, but the manpower and cost for testing can be reduced using model-based simulations. Considering this, Chung et al.^[1] developed a tractor simulation model to reduce ride vibration. A 3D model-based optimal design was produced to minimize the ride vibration. However, the tire characteristics test method and result data need further verification, since the test environment was different from the actual driving conditions and the dynamic characteristics of tires vary widely depending on speed, load, tire inflation, etc. In other words, to predict the exact vibration characteristics by simulation without an actual test, the accuracy of the model needs to be improved by fundamental studies on the components of vibration characteristics in tire models.

Seat suspensions, cabin rubber mounts, cabin suspensions, and rubber tires are the components that most affect the vibration in a tractor system. However, the dynamic characteristics of each component are not distributed by manufacturers and preceding research is insufficient. The tractor cabin rubber mount was recently studied to reduce tractor ride vibration. Choi et al.^[2] experimentally investigated stiffness and the damping coefficient

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of cabin rubber mounts which govern the damping characteristics of cabin vibration. As most commercialized tractors do not have axle suspension, unlike regular passenger cars, vibrations from the road surface are transmitted to the vehicle body directly, affected only by the tires. Because the dynamic characteristics of tires act as a primary vibration reducer and significantly affect the ride comfort, they are considered important for conducting model-based research for needs such as reducing ride vibration and driving simulation. However, little has been studied about the stiffness and damping coefficient of rubber tires so far.

Kim et al.^[3] measured the radial natural frequency of the passenger car tire which is related to the stiffness and damping coefficient and affects the ride vibration. A shock vibration test utilizing an impact hammer as an exciter, forced vibration test utilizing a shaker as an exciter, and drum test was conducted for the measurement. These tests are commonly conducted to measure the vibration characteristics of machine systems, including tires, but they are not practical for tractor tires. In the case of the shock vibration test, the natural frequencies of tires vary with the excitation load and loads caused by the tractor body due to the non-linear behavior of rubber, resulting in varying stiffness and damping coefficients. In addition, it is difficult for the tester to excite the tire in the exact vertical direction using the impact hammer. In contrast, in the case of the forced vibration test and drum test, the diameter of tires for 110 kW-class tractors used in this study is twice as large as the one for passenger cars, making it difficult to design the test jig and drum. Various sizes of jigs and drums should be prepared when the tests are repeated since the size of tractor tires varies widely depending on the size of the tractor. In addition, the equipment should be designed to withstand the large weight of a tractor or high loads. Thus, it is difficult to carry out the forced vibration test or drum test.

Sleeper and Dreher^[4] suggested test methods measure stiffness and damping ratio, where the tests were divided into static and dynamic tests. In the static test, the tire was compressed slowly in the vertical direction while measuring the load and deformation to obtain a hysteresis loop by which static stiffness and damping ratio are determined. The dynamic stiffness and damping ratio were determined by a logarithmic decrement method, utilizing the displacement data from the free-vibration test. Kising and Göhlich^[5] developed a test stand on which the tire can be vibrated by an electro-hydraulic actuator and rolled on the belt track, instead of a drum, for the study of dynamic characteristics of tractor tires. However, the test stand could not represent the actual riding conditions and environment. Lines and Young^[6] developed a test machine connected behind a tractor. The target tire was mounted on the jig and the load was applied by a hydraulic actuator while the tractor was driven on the actual farmland and acceleration data was measured. They analyzed how dynamic behavior varies depending on driving conditions by repeating the test in various conditions. The test result was more realistic than other test methods since it was performed in an actual tractor driving condition. In subsequent research, Lines and Murphy^[7,8] analyzed how the stiffness and damping coefficients are affected by tire inflation, size, age, wear, driving condition, etc. It was concluded, for 10 or more evaluated tractor tires, that the stiffness was 300-500 kN/m and the damping coefficient was 1.5-3.5 kN·s/m. Additionally, a predictive formula for stiffness, using age, size and inflation was suggested.

As previously mentioned, the dynamic characteristics of tires

are not being actively studied. Most previous studies on tractor tires, in particular, had the limitation of lab tests being unable to represent actual driving conditions but were conducted at static condition or unloaded condition. For example, Cuong et al.^[9] and Enlai et al.^[10] conducted drop tests with steel weights to determine the stiffness and damping coefficient. However, the drop test might yield different results as the test was not carried out in the driving condition, and could not reflect the dynamic effect of a rubber tire. Recently, Witzel^[11] performed flat belt tests to determine the stiffness and damping coefficients considering the driving condition and axle load. Additionally, because of the massive size of tractor tires, test jigs and equipment were made for each tractor, resulting in much cost and difficulty of repetition.

In this study, a new driving test method, referred to as a cleat test, was suggested. The cleat test was simpler than conventional methods and conducted under driving conditions so that the results were more feasible. Ground-vertical acceleration was measured while the tractor was driving on the asphalt test track and passing cleats fixed on the track. Based on the measured acceleration, stiffnesses and damping coefficients were determined by the logarithmic decrement method and least squares exponential curve fitting. After that, the dynamic stiffness was compared with the static stiffness, which was determined through a compression test. Finally, the results of the cleat test were verified by comparison with the results of a preceding study.

2 Materials and methods

2.1 Test tractor and tires

This study was conducted on the front and rear tires of a 110 kW-class tractor. The test tractor was a TX1500 (Tong Yang Moolsan, Seoul, Republic of Korea) and the tires were Agrimax RT 855 (Balkrishna Industries Limited, Mumbai, India). The specifications of the tractor are presented in Table 1 and the specifications and weights applied to each tire are presented in Table 2. The weights were measured by placing each tire on the load cells at the same time; they were then inputted to compute the stiffness and damping coefficient of each tire. Directions of each load are presented in Figure 1. The inflation pressure was set to the recommended value of the product catalog.

Table 1 Specifications of the tractor used in this study

| Item | Specification |
|--------------|-------------------------|
| Model | TX1500 |
| Manufacturer | Tong Yang Moolsan (TYM) |
| Power | 110 kW |
| Total Weight | 5528 kg |

Table 2 Tire specifications and applied weight on each tire

| | Front Tire | | Rear Tire | |
|------------------------|--------------------|--------------------|--------------------|--------------------|
| | Left | Right | Left | Right |
| Applied weight /kN | 11.46 (W_{FL}) | 11.60 (W_{FR}) | 15.65 (W_{RL}) | 15.51 (W_{RR}) |
| Size | 380/85 R 28 | | 460/85 R 38 | |
| Overall diameter /mm | 1357 | | 1747 | |
| Section width /mm | 380 | | 475 | |
| Inflation pressure/kPa | 160 | | 160 | |

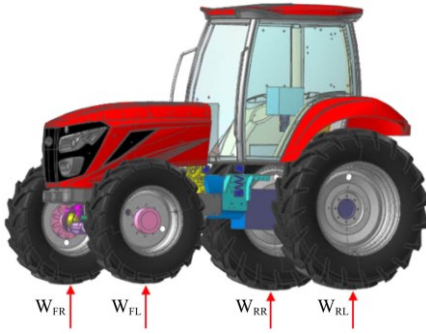


Figure 1 Directions of applied weights

2.2 Theoretical background

2.2.1 Underdamped motion, stiffness and damping coefficient

Most single-direction vibration systems can be represented as a single degree of freedom damped vibration system as in Equation (1). A point contact model^[12], which is the simplest one of various tire modelling methods, was used for vibration system modelling in this study. The point contact model assumes that the spring and damper are placed in parallel, with one point of contact to ground. As the point contact model is a single degree of freedom model, this model can be simply analyzed by applying Equation (1), which describes a general vibration system.

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (1)$$

where, m is mass of the system, kg; c is damping coefficient, kN·s/m; k is stiffness, kN/m; $x(t)$ is the displacement at time t , m.

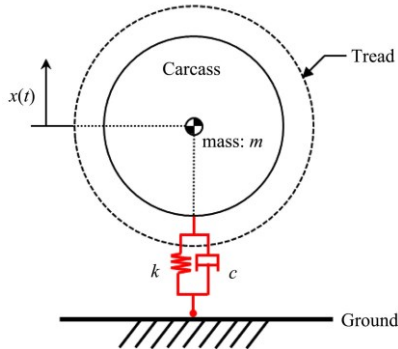


Figure 2 Point contact model of tire

The vibration theory of Inman^[13] was used to analyze the vibration system. If the assumption is made that displacement is expressed as $x(t)=ae^{λt}$, the root of the characteristic equation is derived as Equation (2):

$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2m} \sqrt{c^2 - 4km} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \quad (2)$$

where, ζ is the damping ratio; ω_n is the undamped natural frequency, rad/s. Depending on the sign of c^2-4km or ζ^2-1 , λ becomes a real or complex number, which determines the vibration type of the system. When ζ is larger than 0 and smaller than 1, the amplitude of oscillation gradually decreases after the initial excitation and is called underdamped motion. Underdamped motion is the most commonly observed type in mechanical systems. Tractor tires also show underdamped motion after radial excitation happens. Therefore, the tire vibration system was assumed to be an underdamped system in this study. In underdamped systems, the displacement of the system can be organized as shown in Equation (3):

$$x(t) = e^{-\zeta\omega_n t} (a_1 e^{j\sqrt{1-\zeta^2}\omega_n t} + a_2 e^{-j\sqrt{1-\zeta^2}\omega_n t}) = A e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) \quad (3)$$

where, A and ϕ are constants and ω_d is the damped natural

frequency, rad/s. As Equation (3) has the same form even after it is differentiated to be velocity or acceleration via trigonometric identities, acceleration was measured in this study. The damped natural frequency is defined as Equation (4):

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = \frac{2\pi}{T} \quad (4)$$

where, T is the period of oscillation. The actual vibration of the system is divided into transient vibration, which is caused by external excitation from the impact of the cleat, and steady-state vibration, which is caused by internal components and exists in the periodic form of a sine wave. Equation (3) reflects only transient vibration. The dashed line in Figure 3 corresponds to the decay curve of transient vibration which can be expressed in the form as $Y = Ae^{-\zeta\omega_n t}$.

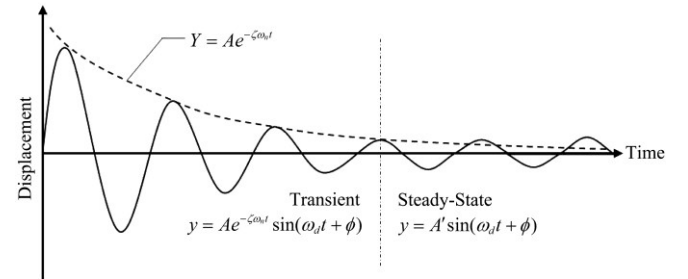


Figure 3 Transient, steady-state vibration and decay curve of underdamped motion

If the vibration period is assumed to be constant, the logarithmic decrement method can be applied to derive vibration characteristics^[13,14]. Logarithmic decrement (δ) is defined as Equation (5):

$$\delta = \ln \frac{x(t)}{x(t+T)} = \zeta\omega_n T \quad (5)$$

From the relationship between damped natural frequency, underdamped natural frequency and the damping ratio, Equation (5) can be converted to Equation (6). The damping ratio and underdamped natural frequency are then computed as Equations (7) and (8):

$$\delta = \ln e^{\zeta\omega_n T} = \zeta\omega_n T = \zeta\omega_n \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{2\pi}{\sqrt{1-\zeta^2}} \quad (6)$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (7)$$

$$\omega_n = \frac{\delta}{\zeta T} = \frac{\sqrt{4\pi^2 + \delta^2}}{T} \quad (8)$$

Utilizing Equations (7) and (8) and definition of stiffness and damping coefficient, stiffness and damping coefficient can be derived as Equations (9) and (10):

$$k = \omega_n^2 m = \frac{m(4\pi^2 + \delta^2)}{T^2} \quad (9)$$

$$c = 2m\omega_n \zeta = \frac{2m\delta}{T} \quad (10)$$

This means that the stiffness and damping coefficient can be determined when mass, period of vibration and logarithmic decrement, which can be calculated by measured acceleration data, are known.

However, owing to the inaccuracy of decay curve fitting, which will be described in the discussion, the damping coefficient was derived from a least squares exponential curve fitting, which reflects multiple peaks of acceleration data, rather than the logarithmic decrement method, which reflects only two peaks.

According to the process of least squares exponential curve fitting, the first-order regression equation of the semi-log decay curve is estimated from the logarithmized acceleration of peaks. The formula of the decay curve can be converted as Equation (11). Defining $-\zeta\omega_n$ as a single constant d , Equation (12) is the logarithmized form of Equation (11).

$$Y = Ae^{-\zeta\omega_n t} = Ae^{dt} \tag{11}$$

$$\ln Y = \ln A + dt \tag{12}$$

Equation (12) can be converted to a simple linear function, in the form of $y=c+dt$. When the linear function is assumed to pass the semi-log of the first peak point (t_1, y_1) , which occurs when the tire is excited as soon as it passes the cleat, the slope of the linear function (d) can be estimated by the below procedure.

Equation (13) is the least squares regression line of $y=c+dt$ with a given set point of (t_1, y_1) :

$$\hat{y}_i = y_1 + \hat{d}(t_i - t_1) \tag{13}$$

The least squares estimator \hat{d} , which minimizes the residual sum of squares (Equation (14)), can be calculated by solving Equation (15), which is the derivation of Equation (14) with respect to \hat{d} :

$$\sum_{i=2}^n e_i^2 = \sum_{i=2}^n (y_i - \hat{y}_i)^2 = \sum_{i=2}^n [y_i - \hat{d}(t_i - t_1) - y_1]^2 \tag{14}$$

$$\sum_{i=2}^n 2(y_i - y_1)(-t_i + t_1) + 2\hat{d}\sum_{i=2}^n (-t_i + t_1)^2 = 0 \tag{15}$$

where, i is number of each peak and n is the total number of peaks.

Hence, \hat{d} is estimated as Equation (16). Consequently, the damping coefficient is newly calculated by Equation (17):

$$\hat{d} = -\zeta\omega_n = -\frac{\sum_{i=2}^n (y_i - y_1)(-t_i + t_1)}{\sum_{i=2}^n (-t_i + t_1)^2} \tag{16}$$

$$C_{regression} = 2m\zeta\omega_n = 2m(-\hat{d}) \tag{17}$$

2.2.2 Force-Deformation curve and static stiffness

Common materials, such as metal, have a linear relationship between load and deformation. The ratio of force (stress) to deformation (strain) of linear materials, which is the slope of Figure 4a, called static stiffness, is constant regardless of the extent of the load. However, the static stiffness of rubber materials differs as the extent of deformation or load is varied, due to the Mullins effect^[15], which makes the rubber behave non-linearly when stress or force is applied. The Mullins effect is shown in Figure 4b. Additionally, static stiffness, according to the study of Amin^[16], is dependent on the rate of imposing loads.

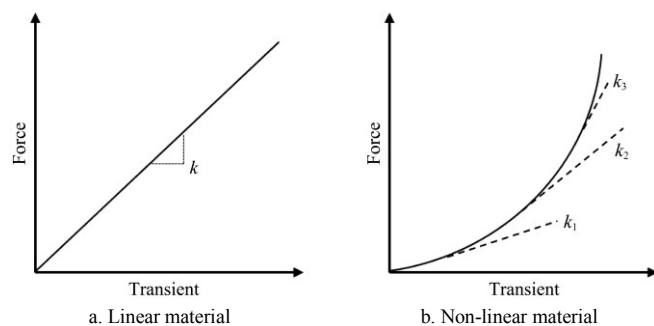


Figure 4 Force-deformation curve of linear and non-linear material

As a tire consists of carcass and tread, in other words, a tire is a combination of linear and non-linear materials, it is necessary to check if stiffness varies when the deformation, load, or compression rate varies. The stiffness of particular deformation of a load can be determined by Equation (18):

$$k_{static} = \frac{dF}{dD} \tag{18}$$

where, F is compression force and D is deformation.

2.3 Test method

In this study, the cleat test was conducted to determine the dynamic stiffness and damping coefficient by the logarithmic decrement method and least squares exponential curve fitting. And compression test was conducted to determine static stiffness by the force-deformation curve.

2.3.1 Cleat test

The cleat test is typically conducted to measure the dynamic characteristics of tires in general vehicles. In this test, accelerometers were glued on each wheel hub and the vertical acceleration was measured when the tires passed the cleat (Figure 5). The acceleration of each tire (Front left, Front right, Rear left, Rear right) was measured at the same time, separately. According to Brinkmann^[17], who applied the cleat test for tractors, the size of the cleat should be minimized so that the impact can be transmitted as quickly as possible, but it should be large enough to at least contact a pair of lugs. In the pretest of our study, the engine of the tractor was shut down with the gear forced to turn into neutral when the height of the cleat was too high (75 mm, 100 mm) or the speed of tractor was too fast (20 km/h). So, 40 mm height cleats were installed and the speed of the tractor was set to 7 km/h and 10 km/h, which are the most common speeds during operation. The cleats adhered to the track as shown in Figure 6. The specifications of sensors, the data acquisition system and GPS sensor are listed in Table 3.

2.3.2 Compression test

The compression test is most commonly used for determining static stiffness. After being fixed as shown in Figure 7, the tire was loaded by a hydraulic compressor while the tire deformation and compression force were measured. To verify whether there was any effect due to compression rate, the test was repeated with different compression rates (3 kg/s, 5 kg/s, 6.67 kg/s, 10 kg/s, 20 kg/s).

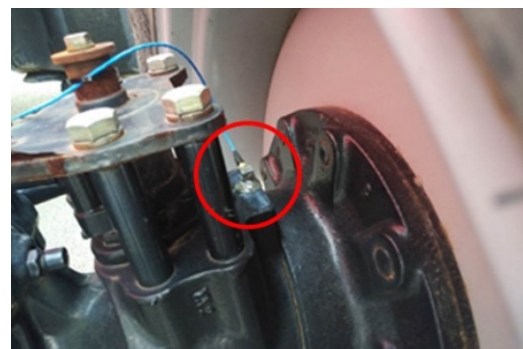


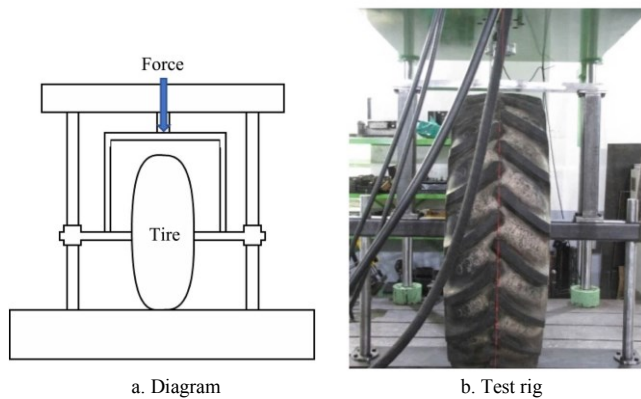
Figure 5 Accelerometer mounted on wheel hub



Figure 6 40 mm cleats attached to the test track

Table 3 Specification of sensors and DAQ used during cleat test

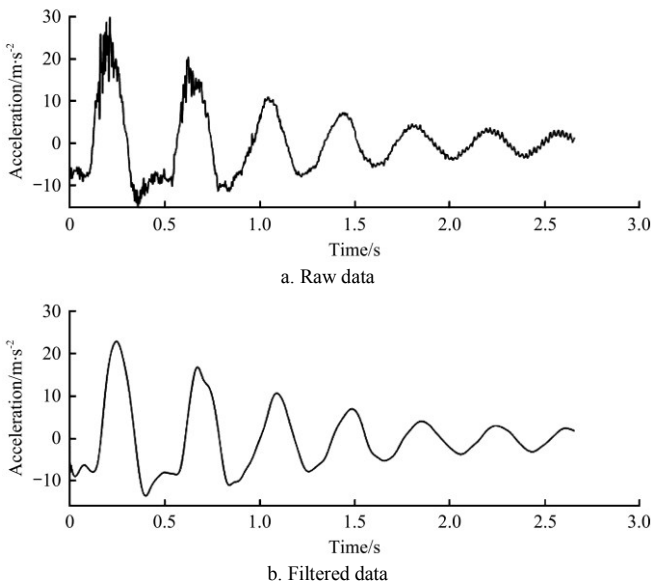
| Equipment | Model | Specification |
|-------------------------|-------------------|---|
| Data acquisition system | HBM eDAQ | Maximum analog channel: 64-96 |
| | | Frequency range: 0.1-10 ⁵ Hz |
| | | Input voltage: 10-55 VDC |
| | | Digital I/O: -0.3 V to 5.5 V |
| Accelerometers | PCB 356A33 | Maximum output pull-up voltage: 5.5 V |
| | | Sensitivity: 10 mV/g (±10%) |
| | | Measurement range: ±4905 m/s ² |
| GPS sensor | HBM EGPS-200 Plus | Frequency range: 2-7000 Hz |
| | | Maximum velocity: 514 m/s |
| | | Minimum velocity: 0.01 km/h |
| | | Accuracy: ±0.05 km/h |



a. Diagram
b. Test rig
Figure 7 Test rig for compression test

2.3.3 Data post-processing

The acceleration data of the cleat test included noise from other sources such as high frequency vibration from the engine or vehicle body or irregular vibration from the track surface as well as excitation from the cleats. To filter out these noises, a low-pass filter was designed based on the Butterworth function of Matlab^[18]. The filter was set to a 5th order and the cut-off frequency was set to 100 Hz. An example of the filtering results is shown in Figure 8.



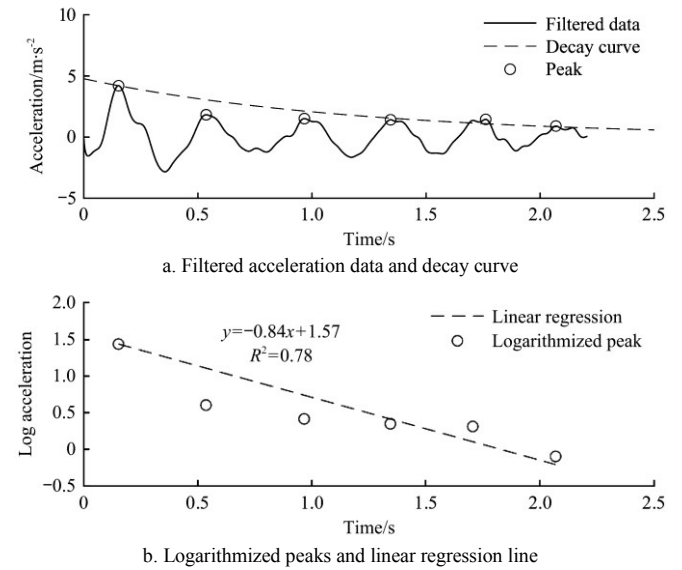
a. Raw data
b. Filtered data
Figure 8 Raw data and filtered data using a Butterworth low pass filter

3 Results

3.1 Cleat test results

Results of the cleat test are presented in Figure 9. In Figure

9a, the solid line represents filtered data and the peaks included in the least squares fitting are highlighted by small circles. The dashed line, representing the decay curve of acceleration, is the exponential regression line of Figure 9b. Stiffness and damping coefficients are calculated and are listed in Table 4 with R-squared values. Also, the variables used to derive the two parameters are listed together. Both parameters are commonly higher in front tires than rear ones at the same speed. The results of the cleat test showed that the front tires have a stiffness of 486.08-570.69 kN/m and a damping coefficient of 4.02-4.52 kN·s/m; and the rear tires have a stiffness of 409.42-483.79 kN/m and a damping coefficient of 2.21-2.67 kN·s/m. The data was well-regressed, with R² values over 0.85 except for RR and RL tire characteristics at 10 km/h.



a. Filtered acceleration data and decay curve
b. Logarithmized peaks and linear regression line
Figure 9 Example of cleat test result (10 km/h, RL tire)

Table 4 Stiffnesses and damping coefficients calculated from cleat test

| Velocity /km·h ⁻¹ | Tire | T/s | ζ | δ | ω _n /rad·s ⁻¹ | K /kN·m ⁻¹ | C _{regression} /kN·s·m ⁻¹ | R ² |
|------------------------------|------|------|------|------|-------------------------------------|-----------------------|---|----------------|
| 7 | FL | 0.29 | 0.15 | 0.95 | 21.65 | 551.33 | 4.26 | 0.92 |
| | FR | 0.31 | 0.09 | 0.62 | 20.27 | 486.08 | 4.02 | 0.95 |
| | RL | 0.39 | 0.09 | 0.59 | 16.02 | 409.42 | 2.22 | 0.85 |
| | RR | 0.38 | 0.03 | 0.16 | 16.47 | 428.80 | 2.65 | 0.91 |
| 10 | FL | 0.29 | 0.15 | 0.95 | 22.03 | 570.69 | 4.21 | 0.86 |
| | FR | 0.29 | 0.15 | 0.98 | 21.75 | 559.80 | 4.52 | 0.87 |
| | RL | 0.38 | 0.13 | 0.83 | 16.53 | 436.22 | 2.67 | 0.78 |
| | RR | 0.36 | 0.06 | 0.39 | 17.49 | 483.79 | 2.21 | 0.70 |

3.2 Compression test results

The compression test results showed that both the front and rear tires showed linearity between force and deformation, as shown in Figure 10. Therefore, static stiffness was estimated as the slope of the force-deformation curve, and the results are shown in Table 5. A significant effect by the compression rate was not verified by this study. The static stiffness of the rear tire was observed to be 15 kN/m higher than the front tire on average.

Table 5 Static stiffnesses from compression test

| | | Compression rate/kg·s ⁻¹ | | | | |
|--------------------------------------|------------|-------------------------------------|--------|--------|--------|--------|
| | | 3 | 5 | 6.67 | 10 | 20 |
| Static stiffness /kN·m ⁻¹ | Front tire | 275.65 | 291.28 | 295.08 | 295.83 | 297.19 |
| | Rear tire | 305.74 | 322.45 | 292.10 | 308.74 | 308.10 |

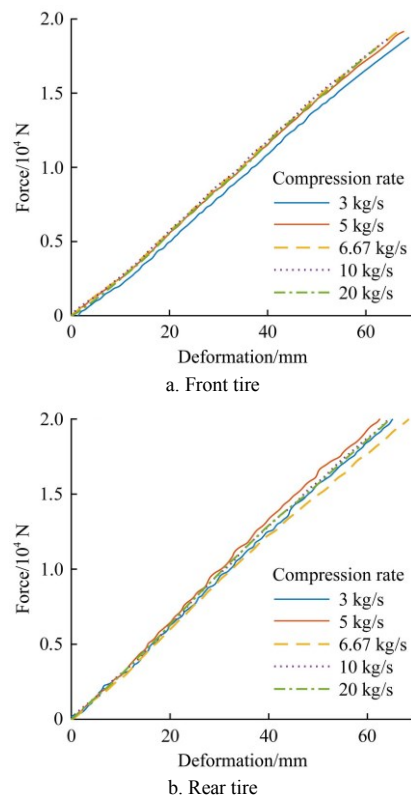


Figure 10 Force-deformation curve attained by compression test

4 Discussion

Table 6 shows the results of the cleat test, static stiffnesses determined by the compression test and results from the study of Lines and Murphy^[7,8] using a field test machine and force vibration test. Lines and Murphy conducted their study by a field test, using similar sizes of tires. Therefore, it is beneficial to compare the results with the cleat test.

Table 6 Stiffnesses and damping coefficients from cleat test, compression test and forced vibration test of Lines and Murphy

| | Stiffness/ $\text{kN}\cdot\text{m}^{-1}$ | Damping coefficient / $\text{kN}\cdot\text{s}\cdot\text{m}^{-1}$ |
|-----------------------|--|--|
| Cleat test | 409.42-570.69 | 2.21-4.52 |
| Compression test | 275.65-322.45 | - |
| Forced vibration test | 300-500 | 1.5-3.5 |

In the case of the rear tire, the ranges of the stiffness and damping coefficients from the cleat test were included in the reference data from Lines and Murphy. However, in the case of the front tire, both parameters were larger than the reference data. It is difficult to determine which test method is better because obvious differences exist in the tire specification, test environment and method of analysis. Nonetheless, the results of the cleat test are reasonable, compared with the forced vibration test. The dynamic stiffnesses from the cleat test were about 100-170 kN/m larger than static stiffnesses from the compression test. To find the difference in the results of several test methods, Lines and Young^[6] conducted 3 kinds of dynamic tests (apparent method, free vibration, forced vibration) and static compression test. Each dynamic test resulted in dynamic stiffnesses of 450 kN/m , 485 kN/m , 480 kN/m , whereas the compression test resulted in static stiffnesses of 330 kN/m , 120-155 kN/m smaller than the dynamic stiffnesses. According to their study, stiffness is lower in compression tests than in dynamic tests because the load is slowly applied to eliminate dynamic effects, resulting in creeping from rubber tire's hysteresis characteristic. Additionally, it is

dependent on the loading and unloading rate. Nang^[19] reported the static stiffness from a compression test was 15%-20% lower than the dynamic stiffness from a drop test. These results are in line with our result, explaining the reason for lower static stiffness than dynamic stiffness. Therefore, the dynamic stiffnesses and damping coefficients from the cleat test should be input to a tractor ride vibration model to represent the dynamic characteristics of rubber tires, other than using static stiffness.

While analyzing the data of the cleat test, it was found that errors are present for the same driving speed and same tire. There were two main sources of error. The first one is the feasibility of exponential curve fitting. Theoretically, the decay curve, in underdamped motion, must be tangent to the acceleration. However, in practice, it was not possible to fit the decay curve tangent to the acceleration peaks because of irregular vibrations from the track, engine or inner parts of the vehicle and mutual vibrational interference between each tire when passing through the cleat. For optimum replication of the vibration system, the least squares exponential curve fitting method was applied to determine the damping coefficient, instead of the logarithmic decrement method, which fit the decay curve using only two peaks. To minimize the error, the curve was fixed to pass the first peak, which occurs when the tires pass through the cleat, and reflect the other peaks by regression. However, stiffness could not be derived by using this method directly. At least one of ζ or ω_n should be known to estimate it. Thus, the decay curve fitting algorithm should be studied further, noting the fact that test results can be impractical or differ with curve fitting methods and that stiffness cannot be estimated by the least squares fitting method.

The second source of error is the superposition of vibrations of each tire and irregular vibration. In Figure 11, the dashed rectangular box indicates the peaks caused when rear tires are passing through the cleat, after the front tires reached the peaks. In this case, the shape of the decay curve and values of the characteristic coefficients vary with sensitivity depending on which peak is included in the regression. As mentioned above, the tractor was driven at a speed of 7 km/h and 10 km/h (1.94 m/s and 2.78 m/s) in the test. Since the wheelbase of the tractor was 2.77 m , the front and rear tires were assumed to have intervals of 1.43 s and 1 s to pass through the cleat at each speed, respectively. Based on this assumption, the peaks that occurred at 1.43 s and 1 s after the first peak were not included in regression when fitting the decay curve of the front tires. Because of this limitation, only 3 or 4 peaks were included in regression of the front tires' decay curve fitting; whereas at least 5 peaks were included for rear tires. Additionally, there were relatively small peaks or inflected vibrations (dotted triangles in Figure 11) 0.01 s before or after the main peaks (circles in Figure 11). These were assumed to have occurred because of the mutual vibrational interference between the left and right tires' cleat impact and vibrational noises (i.e., uneven road surfaces, vibration from engine or transmission) other than excitation from the cleat. These peaks might have influenced the peaks' time and extent of acceleration.

In the case of the results of rear tires under the 10 km/h condition, the estimated R-squared values were under 0.8. In most cases, the acceleration of the second peak could not reach the fitted decay curve, and the gap was significantly bigger in the rear tires. This makes the R-squared values lower, resulting in inadequate stiffness, which is estimated by the first and second peaks. Therefore, further consideration is needed on how to increase the reliability of the analysis results.

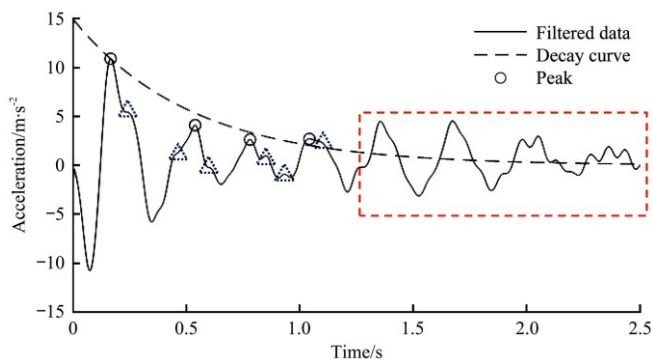


Figure 11 Vibration affected by other tires (triangle: between left and right tire, rectangular: between front and rear tire)

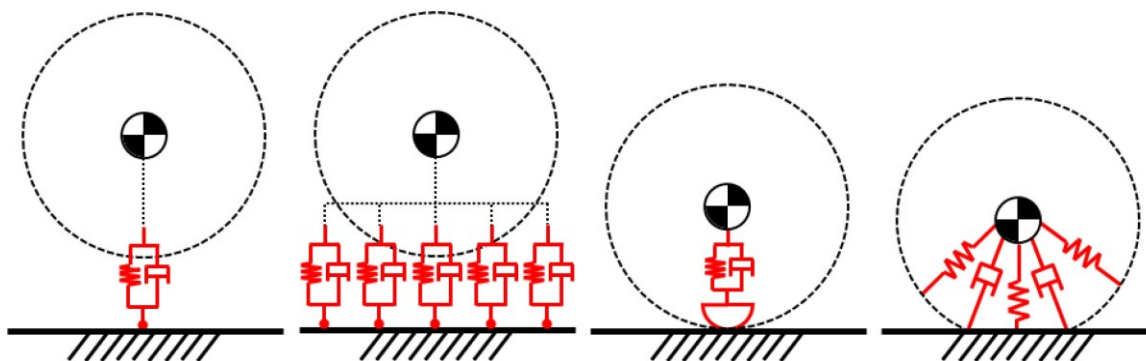


Figure 12 Various tire models^[22] (point contact model, fixed footprint model, rigid tread band model, radial spring or adaptive footprint model from left)

In the future, it is necessary to input stiffness and damping coefficients based on the cleat test and Lines' study into dynamic simulations of tractors and watch the result to verify which values describe the actual vibration system well.

5 Conclusions

In this study, a reliable and simple test method of measuring stiffness and damping coefficients was suggested to improve the accuracy of tractor simulation. Conventional test methods require test equipment for each tire, such as test jigs and shakers, making it difficult to conduct tests and costly. Based on this limitation, the cleat test was conducted to simplify the test procedure, reduce costs, and reflect the actual driving conditions needed to estimate dynamic characteristics. The test was performed by measuring the vertical acceleration of a tractor tire axle using an accelerometer as it passed through a 40 mm height cleat. Stiffnesses were estimated from the first two peaks of acceleration, using the logarithmic decrement method, and damping coefficients were estimated by using least squares exponential curve fitting, which reflects the effects of at least 5 peaks of acceleration.

The test result showed that the front tires have stiffnesses of 486.08-570.69 kN/m and damping coefficients of 4.02-4.52 kN·s/m; and the rear tires have stiffnesses of 409.42-483.79 kN/m and damping coefficients of 2.21-2.67 kN·s/m. However, further studies and verification are needed because the data varies with the decay curve fitting method, test conditions, test environment, tire modelling, etc. The measured decay curve of acceleration was different from the theoretical one. Mutual vibrational interferences between the tires' cleat impact and noises from the road surface and inner excitation were the primary reason for the discrepancy, resulting in deviation of the stiffness and damping coefficient.

Nonetheless, this study is meaningful in that it has presented a

Aside from the analysis procedure, the tire model should be reconsidered. In this study, the point contact model was applied. However, the tire faces the ground due to the deformation of rubber caused by the weight of vehicle. To reflect this, various models were developed as shown in Figure 12. Bernard et al.^[20] found that the vertical load and deformation of tires were significantly different when the point contact, fixed footprint and radial spring models were applied. Crolla et al.^[21] reported that the series model of spring and damper predicted longitudinal and lateral acceleration of tractor tires better than the parallel model. In short, the extent of stiffness and the damping coefficient differs based on what tire model is used. Hence, it is necessary to compare the results calculated from different tire models and to use the optimal one.

new method that improved the practicality of results, reduced cost and simplified the test process for measuring stiffnesses and damping coefficients of rubber tractor tires.

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