Spline regression modelling of PTO performance of tractor fuelled with different biodiesels

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Abstract: The objective of this study was to investigate the possibility of fitting spline regression models for power take-off (PTO) performance characteristics of an agricultural tractor tested with four different fuels, including diesel fuel (B0) and three biodiesel blends made of canola oil (B10: 10% biodiesel + 90% petro-diesel blend; B20: 20% biodiesel + 90% petro-diesel blend; B30: 30% biodiesel + 90% petro-diesel blend). The performance characteristics evaluated were PTO power, engine torque, engine fuel consumption, and specific fuel consumption. Due to sharp slopes in measured quantities around the nominal engine speed (2200 r/min), compared to the standard regression method, the spline regression models suited well to the experimental data with coefficient of determination R^2=0.99 for PTO power and engine torque. R^2 varied between 0.97 and 0.99 for fuel consumption and 0.91 and 0.95 for specific fuel consumption. The weaker correlation found for specific fuel consumption could be attributed to profound fluctuations in measured data causing atypical pattern in the corresponding graphs around the nominal engine speed. It was concluded that splines were useful to accurately predict the measured PTO power and engine torque. Neither standard methods nor splines were sufficient to obtain very good regression models for specific fuel consumption.
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1 Introduction

The tractor is known to be the most important power source among all agricultural machines[1]. Although the main energy source of agricultural tractor engines is conventional diesel fuel, renewable energy sources such as biodiesel are also utilised as an alternative energy source for engines due to the possibility of depletion of oil reserves in the future[2].

Tractor engine tests provide information on the performance characteristics including engine power, power take-off (PTO) torque, fuel consumption, and specific fuel consumption as a function of engine rotational speed. These characteristics are important to compare performance characteristics of tractor engines. The graphical representation of performance characteristics is considered sufficient to show the technical features for a given tractor. Engine tests are conducted in accordance with the engine test procedures described by OECD Test Code 2[3] and it is customary to display test data in graphical forms.

When the use of biodiesel blends is considered, most

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attention has been paid to measuring engine characteristics and emission values of diesel engines since these characteristics are affected by biodiesel use on engines[4-9]. On the other hand, some studies focused on other issues, such as modelling the combustion processes and emissions[10]. Engine characteristics and emissions were studied for a variety of bio-fuels including cooking mixture oil[11], feedstock oil[12], rapeseed oil[13], canola oil[14], and others[15,16]. These studies were conducted on single or multiple cylinder engines using different biodiesel blends with diesel fuel and showed that different blends of biodiesel fuels could be used on diesel engines as alternative fuels. It was also suggested that the use of oils as fuels could be environmentally friendly and compatible[13].

There were rare literatures reporting mathematical models that relate the tractor engine performance characteristics to the engine speed under various test conditions. The objective of this study was to investigate the possibility of obtaining good mathematical models that will regress engine performance characteristics versus engine rotational speed with applicable coefficients of determination.

2 Materials and methods

To achieve the objective the study, the data used for modelling were taken from Aybek et al.[14] who determined four performance characteristics (PTO power, engine torque, fuel consumption, and specific fuel consumption) of an agricultural tractor (Massey Ferguson 3056 2WD), using conventional diesel (B0), and three different biodiesel blends (B10:10% biodiesel + 90% petro-diesel blend; B20:20% biodiesel + 90% petro-diesel blend; B30:30% biodiesel + 90% petro-diesel blend) from canola oil. The tractor had a power rating of 43 kW, mass of 2223 kg, and nominal engine speed of 2200 r/min. The means and standard errors were calculated for each parameter in the mentioned previous study.

Based on preliminary curve fittings, it was observed that linear regression methods were insufficient for obtaining suitable regression models due to sudden changes in the slopes of the graphs evaluated. Significant variations in measured quantities, particularly at and above nominal engine rotational speed, resulted in poor fit for the given data sets. Also, higher order polynomials may result in fluctuations between the data points while providing good estimates at the points. Splines can be used to eliminate oscillations in fitted data and fits a smooth curve to the points. Although spline can be of any degree, the most common is the cubic splines that have the same slope and curvature at the points where they join, resulting in much better fit[17]. Spline regression model brakes the regression lines into several line segments, resulting in better fits compared to conventional regression methods[18,19]. Thus cubic spline models were developed to study the performance characteristics with the use of different fuels. Appropriate knot points were determined to apply cubic spline regression. Thirteen data points were used to do the data fittings for each of the engine characteristic evaluated in the study. The best fit was sought for four parameters (PTO power, engine torque, fuel consumption, and specific fuel consumption) with four different fuels (conventional (pure) diesel and three biodiesel blends (B10, B20, and B30)) resulting in 16 regression equations. Durbin Watson Autocorrelation (DWA) test was used to determine whether a correlation exists between successive errors and assumes a normal error distribution[18,20].

3 Results and discussion

Linear regression models and polynomials resulted in poor coefficients of determination (R^2=0.1-0.6) for four fuels evaluated, but not presented in detail in this study. The linear models were inappropriate due to the unusual slopes in the graphs of the performance characteristics. Thus an attempt was made to use splines to obtain more useful regression functions. The autocorrelations of models were tested using Durbin-Watson *t*-test.

The spline regression curves fitted to the data are given in Figures 1-4. Coefficient of determination (R^2), sum of squared errors (SSE), Durbin-Watson (DW), Akaike Information Criterion (AIC) values, and degree of freedom (DF) for the regression equations are presented in Tables 1-4, and the corresponding mathematical models found for the performance characteristics are given in Equations (1)-(16).

Expressions in Equations (1)-(4) can be used to estimate the PTO power as a function of engine rotational speed. The spline curves matched the experimental data well for PTO power for the four fuels (Figure 1). The differences in the curves were usually small and visually indistinguishable. Summary statistics of spline regression suggested that the PTO power could be estimated as a function of engine speed with R^2=0.99 for the four fuels.

The engine speed providing the greatest PTO power is known as the nominal engine speed and is an important technical specification for an engine. The maximum PTO power was 32.23 kW at an engine speed of 2200 r/min using the conventional diesel whereas the maximum PTO power was accomplished at 2100 r/min for all biodiesel blends[14]. The cubic spline regression modelling of PTO power data showed that each fuel blend had similar behaviour, i.e. the line segments on the rotational engine speed scale were similar (Figure 1). The spline curves did not go through the measured data point at 2200 r/min due to sharp decrease in measured quantities above the nominal engine speed. Despite the discrepancies between the measured values and the estimated points by spline equations, spline regression models (Equations (1)-(4)) provided good fittings with high coefficients of determination. Statistical analysis suggested no autocorrelation for PTO power and engine torque data (Tables 1 and 2), implying that the best possible regression functions have been chosen. The functions given in Equations (1)-(4) correspond to the fuels B0, B10, B20, and B30, respectively:

$$f_1(x) = –45.115 + 0.123x – 7.94(10^{-5})x^2 + 2(10^{-83} – 5(10^{-5})(x – 1500)^3 – 17.3(10^{-5})(x – 2200)^3 \quad (1)$$

$$f_2(x) = –32.31 + 0.100x – 6.6(10^{-5})x^2 + 1.67(10^{-8})x^3 – 4.6(10^{-8})(x – 1500)^3 – 1.7(10^{-5})(x – 2200)^3 \quad (2)$$

$$f_3(x) = –19.402 + 0.073x - 4.6(10^{-5})x^2 + 1.2(10^{-s})x^3 – 4.05(10^{-8})(x – 1500)^3 – 1.7(10^{-5})(x – 2200)^3 \quad (3)$$

$$f_4(x) = 10.409 + 0.0065x – 2.62(10^{-6})x^2 + 2.13(10^{-10})x^3 – 2.64(10^{-8})(x – 1500)^3 – 1.7(10^{-5})(x – 2200)^3 \quad (4)$$

where, x is engine rotational speed, r/min; $f(x)$ is PTO power, kW. Similarly Equations (5)-(8), (9)-(12), and (13)-(16) also correspond to the fuels B0, B10, B20, and B30, respectively.

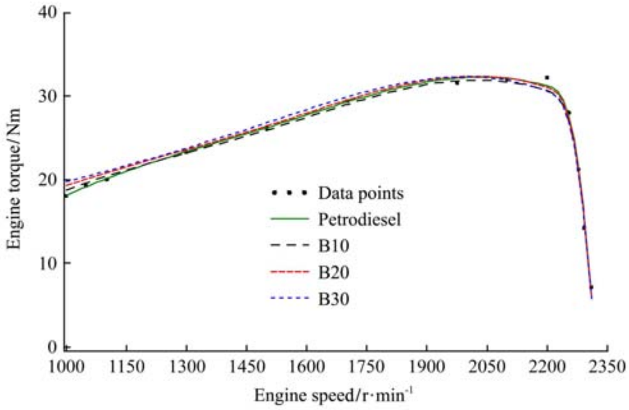

Figure 1 Splines for PTO power as a function of engine speed

Table 1 Summary statistics for spline regression of PTO power

Summary Statistics	Petrodiesel	B10	B20	B30
R^2	0.9911	0.9913	0.9910	0.9910
SSE	6.157	5.77	6.137	6.079
DW	2.176	2.136	2.169	2.155
AIC	39.17	38.33	39.13	39.01
DF	7	7	7	7

Note: Durbin Watson (α=0.01).

The splines fitted to the engine torque data are shown in Figure 2, summary statistics are given in Table 2, and the corresponding mathematical models are given in Equations (5)-(8). The fitting was good (R^2=0.99) for engine torque data for each of the four fuels tested (Table 2). Therefore spline regression was found to be useful to accurately predict the PTO power and engine torque for any given fuel type.

$$f_5(x) = –60.175 + 0.547x – 4.1(10^{-4})x^2 + 1(10^{-7})x^3 – 2.09(10^{-7})(x – 1500)^3 – 7.1(10^{-5})(x – 2200)^3 \quad (5)$$

$$f_6(x) = 145.96 + 0.133x – 1.4(10^{-4})x^2 + 3.98(10^{-8})x^3 – 1.53(10^{-7})(x – 1500)^3 – 6.9(10^{-5})(x – 2200)^3 \quad (6)$$

$$f_7(x) = 149.38 + 0.136x – 1.4(10^{-4})x^2 + 4.04(10^{-8})x^3 – 1.55(10^{-7})(x – 1500)^3 – 7(10^{-5})(x – 2200)^3 \quad (7)$$

$$f_8(x) = 230.07 – 0.048x – 7.3(10^{-7})x^2 + 4.77(10^{-9})x^3 – 1.06(10^{-7})(x – 1500)^3 – 6.8(10^{-5})(x – 2200)^3 \quad (8)$$

where, x is engine rotational speed, r/min; $f(x)$ is engine torque, N·m.

The spline curves, summary statistics, and the regression equations for fuel consumption data of the tractor are given in Figure 3, Table 3, and Equations (9)-(12), respectively. The coefficients of determination were slightly lower compared to PTO power and engine torque models, but may be considered good with R^2=0.97–0.985.

$$f_9(x) = -12.223 + 0.034x - 2.1(10^{-5})x^2 + 5.34(10^{-9})x^3 - 1.44(10^{-8})(x-1500)^3 - 5.43(10^{-6})(x-2200)^3 \quad (9)$$

$$f_{10}(x) = 29.86 - 0.062x + 0.509(10^{-5})x^2 - 1.24(10^{-8})x^3 + 1.25(10^{-8})(x-1500)^3 - 5.27(10^{-6})(x-2200)^3 \quad (10)$$

$$f_{11}(x) = -7.13 + 0.0226x - 1.2(10^{-5})x^2 + 3.01(10^{-9})x^3 - 1.03(10^{-8})(x-1500)^3 - 4.5(10^{-6})(x-2200)^3 \quad (11)$$

$$f_{12}(x) = -13.88 + 0.039x - 2.6(10^{-5})x^2 + 6.44(10^{-9})x^3 - 1.6(10^{-8})(x-1500)^3 - 5.1(10^{-6})(x-2200)^3 \quad (12)$$

where, x is engine rotational speed, r/min; $f(x)$ is fuel consumption, L/h.

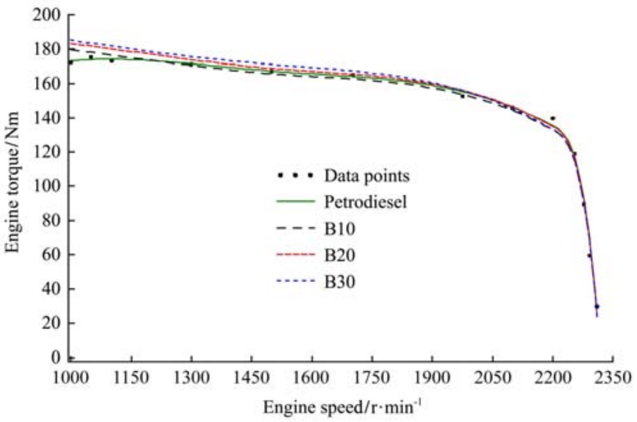

Figure 2 Splines for engine torque as a function of engine speed

Table 2 Summary statistics for spline regression of engine torque

Summary Statistics	Petrodiesel	B10	B20	B30
R^2	0.995	0.996	0.996	0.996
SSE	117.61	107.43	114.12	106.65
DW	2.203	2.175	2.233	2.238
AIC	77.52	76.34	77.13	76.25
DF	7	7	7	7

Note: Durbin Watson (α=0.01).

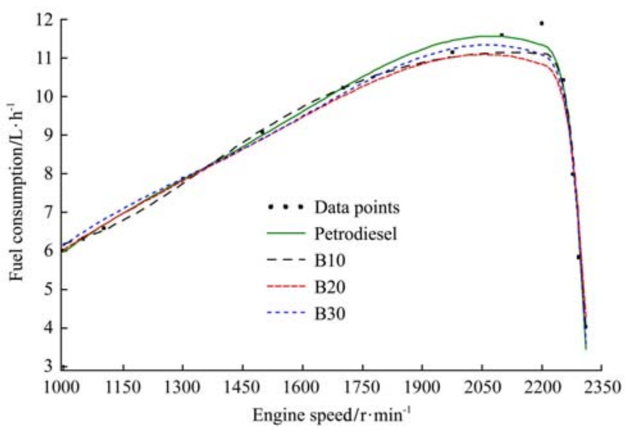

Figure 3 Splines for fuel consumption as a function of engine speed

Table 3 Summary statistics for spline regression of fuel consumption

Summary Statistics	Petrodiesel	B10	B20	B30
R^2	0.980	0.979	0.969	0.985
SSE	1.526	1.352	1.816	1.022
DW	1.916	1.980	1.82	2.05
AIC	21.06	19.479	23.304	15.83
DF	7	7	7	7

Note: Durbin Watson (α=0.01).

The smallest coefficients of determination were found in the case of specific fuel consumption among the four performance characteristics evaluated in the study (Table 4). This was possibly due to the notable fluctuations in measured data and the resultant rapid changes in the slopes in Figure 4 about the nominal engine speed. The measured data points eventually caused the most troublesome pattern among the four characteristics, resulting in an inappropriate model equation for specific fuel consumption. The spline regression equations are given in Equations (13)-(16).

$$f_{13}(x) = -328.85 + 1.41x - 0.0011x^2 + 2.83(10^{-7})x^3 - 5.96(10^{-7})(x-1500)^3 + 1.39(10^{-4})(x-2200)^3 \quad (13)$$

$$f_{14}(x) = 700.81 - 0.99x + 7.41(10^{-4})x^2 - 1.74(10^{-7})x^3 + 1.07(10^{-7})(x-1500)^3 + 1.78(10^{-4})(x-2200)^3 \quad (14)$$

$$f_{15}(\mathrm{x}) = -1096 + 3.107x - 0.0023x^2 + 5.79(10^{-7})x^3 - 1.06(10^{-6})(x-1500)^3 + 2.35(10^{-4})(x-2200)^3 \quad (15)$$

$$f_{16}(\mathrm{x}) = -1274 + 3.5x - 0.0026x^2 + 6.41(10^{-7})x^3 - 1.02(10^{-6})(x-1500)^3 + 1.59(10^{-4})(x-2200)^3 \quad (16)$$

where, x is engine rotational speed, r/min; $f(x)$ is specific fuel consumption, g/kW·h.

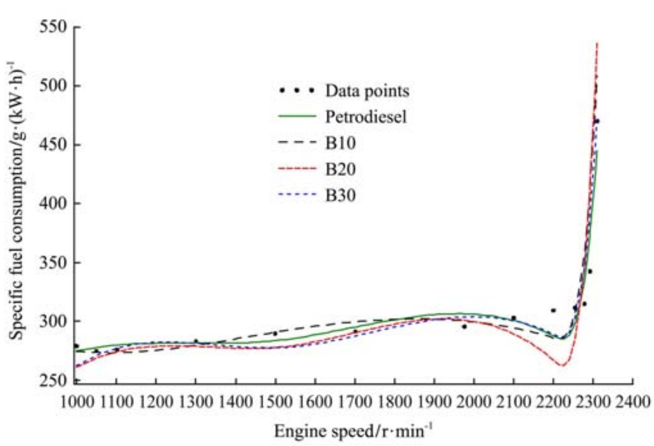

Figure 4 Splines for specific fuel consumption as a function of engine speed

Table 4 Summary statistics for spline regression of specific fuel consumption

Summary Statistics	Petrodiesel	B10	B20	B30
R^2	0.908	0.946	0.907	0.941
SSE	2940.6	3290.5	7656.9	2725.17
DW	1.81	1.91	1.81	1.94
AIC	119.37	120.83	131.81	118.38
DF	7	7	7	7

Note: Durbin Watson (α=0.01).

When AIC values were evaluated for a performance characteristic, the smallest AIC value corresponded to the greatest R^2 in the tables except for Table 4. Despite the small differences in the coefficients of determination in Table 1 or Table 2, there was no difference in the goodness of the fit since all coefficients of determination were practically the same in a single table, implying a good fit was found regardless of the fuel type. Aybek et al.[14] found that B20 was the most appropriate fuel since the use of B20 resulted in the highest PTO power and engine torque and the smallest specific fuel consumption. It should be noted that the spline models found in this study could not be used to determine the best fuel to be chosen, rather the models help determine which performance characteristic could be modelled with good fit.

The results were convincing that splines could be safely used to obtain good fit for the point distribution of the PTO power data and the engine torque data for each fuel tested. Fuel consumption was also accurately estimated as a function of engine speed. The R^2 for specific fuel consumption, however, was not as good as other tractor performance characteristics. Spline regression models break the regression line into several line segments and thus results in regression models with higher R^2 compared to other regression methods. Despite this strength of spline modelling, the value of R^2 was not sufficiently high for specific fuel consumption data. Consequently, it was impractical to derive a regression equation that could estimate the specific fuel consumption as a function of rotational engine speed with a R^2 as high as PTO power or engine torque.

It should be recalled that the engine performance tests were conducted under laboratory conditions as described by OECD Test Code 2[3]. Hence the measured data for each test parameter would not vary significantly based on test conditions if the tests were to be repeated in another study. The properties of the fuels produced, however, might slightly vary depending on various factors, such as fuel processing technology and the type of raw material the biodiesel is made of. As a result, the point distribution of measured data for each performance characteristic might be slightly different in another study owing to the potential changes in fuel quality and fuel properties. The underlying shapes of the curves depicted in Figures 1 to 4, however, would not be different.

Although the findings of this study did not suggest that the regression functions found herein were universally applicable mathematical models, the high coefficients of determination proved the appropriateness and applicability of regression models to relate the dependent and independent variables under the specified test conditions. The estimated spline models could hence be used to determine the point distribution and the mean values of the dependent variables (engine performance characteristics) as a function of the independent variable (engine rotational speed). Despite the complexity of the spline models compared to simpler regression models frequently encountered in many applications, such regression functions could easily be incorporated into software development when needed.

4 Conclusions

1) It is customary to represent the engine characteristics of tractors in graphical form and is not likely to find mathematical expressions that relate the engine speed to the engine characteristics under consideration. This study suggested that engine characteristics could be modelled using spline regression functions and reported high levels of R^2 for the characteristics evaluated.

2) The shapes of splines corresponding to each engine characteristic were similar for pure diesel fuel (B0) and the biodiesel blends (B10, B20, B30). As a consequence, similar coefficients of determination were found with the use four different fuels for each tractor engine characteristic.

3) Spline regression modelling was useful since the relationship between the dependent (PTO power and engine torque) and independent (rotational engine speed) variables was explained effectively with R^2=0.99. No autocorrelation was found, verifying that the best models were found for the data sets.

4) The R^2 was slightly lower (R^2=0.97–0.99) for fuel consumption whereas it further reduced (R^2=0.91–0.95) in the case of specific fuel consumption. It was not possible to regress the engine speed with the specific fuel consumption as effective as other three cases.

5) Although the estimated regression functions are somewhat complex compared to most regression equations, these mathematical expressions could easily be used in software development when needed.

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