Control design of an unmanned hovercraft for agricultural applications

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Abstract: The efficient and precise application of agricultural materials such as fertilizer or herbicide can be greatly facilitated by autonomous operation. This is especially important under difficult conditions at remote sites. The purpose of this work is to develop an accurate nonlinear controller using a direct Lyapunov approach to ensure stability of an unmanned hovercraft prototype used for the execution of these agricultural tasks. Such a craft constitutes an underactuated system which has fewer actuators than degrees of freedom. The proposed closed loop system is simulated to demonstrate that a control law can stabilize both the actuated and unactuated degrees of freedom of the hovercraft. It is shown that the position and orientation of the hovercraft achieve high dynamic and steady performance.

Keywords: controller, direct Lyapunov method, nonholonomic constraint, underactuated system, unmanned hovercraft
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1 Introduction

The use of unmanned vehicles (UVs) for precision agriculture is a field that continuously spurs interest due to the assistance that UVs could provide in reducing the cost and assuring the safety of many agricultural procedures. Given this motivation, a new vehicle is needed to support agriculture, specifically in applications where operator safety is a significant requirement. These applications involve weed control, crop preparation, and efficient use of fertilizers. All of these applications require extensive production capabilities, capabilities that might be especially costly in countries where labor shortages are currently an issue[1].

The vehicle selection must be made considering the environmental risk, the recommended pesticide dose, and the risk of operator exposure. These requirements can be met through the use of an autonomous hovercraft. By using an autonomous hovercraft, the dispersion of the pesticide and fertilizer will have higher effectiveness and will be less costly compared to an aerial vehicle due to the surface proximity on which the vehicles act[2].

A new technology of spraying pesticides and delivering fertilizer which is done in a way that does not disturb the crops will be tested using a hovercraft prototype under agricultural field-conditions[3]. The goal of this effort is to demonstrate the beneficial nature of this approach as a solution to human and crops protection. This paper deals with the first step, to develop means to control the hovercraft so that it is a stable system. Successful completion of the first step would allow a second step consisting of the construction of a prototype to provide a platform for structured work involving agricultural applications and tests to confirm the results of the first step. Figure 1 shows the hovercraft prototype design views and Figure 2 shows the unmanned hovercraft prototype spraying.

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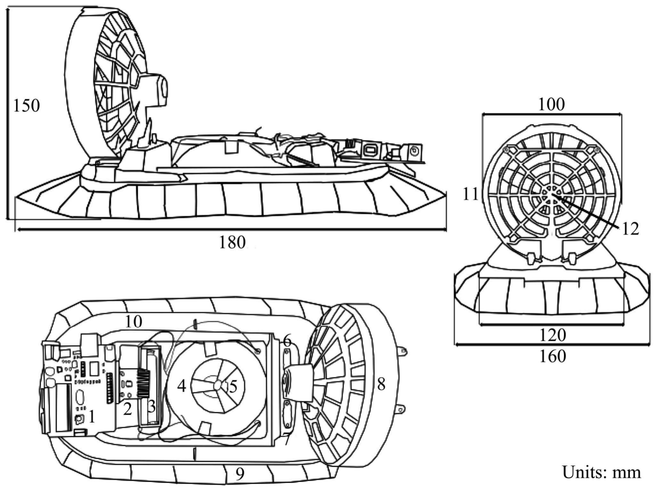


1. Arduino　2. GY-521　3. Battery　4. Skirt motor case　5. Skirt motor　6. Skirt motor switch　7. Propeller motor switch　8. Propeller motor case　9. Skirt　10. Hovercraft assembly　11. Propeller motor grid　12. Propeller motor

Figure 1　Hovercraft prototype design views

Figure 2　Unmanned hovercraft prototype

The hovercraft consists of fans and a cushion where the air pressure inside the cushion enables it to float and move smoothly on any surface. The lift fan is capable of operating for long periods of time and provides the internal cushion pressure. The pressure inside the cushion needs to be maintained at all times in different climates to ensure the hovercraft is free to move. Furthermore, the unmanned hovercraft has lower friction opposing its movement, where the skirt contacts the wet or dry surface, compared to other land or water forms of transportation. This UV can also be propelled over different types of crops without damaging them whereas other vehicles cannot[4]. Some disadvantages when using a hovercraft are that they require a lot of air for lift, are loud due to fan or propeller rotation during the operation, and have power requirements for a particular agricultural application. Because there is no human operator, the audible noise disadvantage does not present an obstacle. In addition, the hovercraft has the potential to damage its skirt or cushion.

The main challenge when designing controls for underactuated systems is the non-linearity of the equations of motion that govern the dynamics together with the manipulation of those equations so that a controller can be found. The application of any method is, in general, a rather difficult task because developing the needed controller involves solving ordinary and partial differential equations. Generally, control strategies for the stabilization of underactuated systems can be found in the literature. Some of the previous studies conducted by several researchers on stabilizing the underactuated, unmanned hovercraft system are mentioned and analyzed below.

In the hovercraft modeled by Marconett[5], the design consists of one powerful hovering motor and four horizontally mounted propulsion motors. A microcontroller acquires input data from various sensors and provided output signals to vary the speed of each motor and performs the necessary stabilization using a proportional-integral-derivative (PID) controller.

The nonholonomic (nonintegrable constraints), autonomous underactuated underwater vehicle (AUV) modeled in literature [6] consists of a control that regulates the dynamic model in the horizontal plane with a desired orientation (roll, pitch, and yaw) using a Lyapunov-based, adaptive formulation. A discontinuous, adaptive state feedback controller is derived that yields convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty.

In the work of Fantoni et al.[7,8], two different control strategies were designed for stabilizing the surge (linear), sway (linear), and angular velocities of the hovercraft frame. The authors used a Lyapunov formulation with the surge force and the angular torque as outputs of the controller. In addition, the mathematical model of the hovercraft system was derived based on Newton's second law and an Euler-Lagrange formulation.

Chaos et al.[9] used a cascade control problem (two loops) to control a vehicle with very simple propeller speed regulation that produced only a discrete set of control commands, based on minimal information of the dynamics, to the actuators. To control the hovercraft, the outer loop stabilizes the position error, and the inner loop stabilizes the orientation of the vehicle. The stability and robustness of the controller is demonstrated in the presence of disturbances and noise through simulation.

Wang et al.[10] used a nonlinear control in order to study an amphibious hovercraft. Here the hydrodynamic and aerodynamic coefficients based on the angular speed and orientation were considered. They introduced an adaptive multiple model approach to acquire a linearized model of the hovercraft and from there to set the different parameters based on weighting methods.

Lindsey[11] modeled a remote controlled (non-autonomous) hovercraft using Newton's second law, where the hovercraft had two thrust and lift fans providing two separate sources of actuation. The open and closed loop behavior of the system was simulated using the Matlab/Simulink environment. The author mentioned that the model was successfully and accurately controlled.

The previous work using the Direct Lyapunov Approach (DLA)[12-15] is taken as the starting point of this formulation for the design of the stabilizing nonlinear control law for underactuated hovercraft systems. The attractiveness of the DLA used in the formulation is that this method offers a wider range of applications and the obtained linear algebraic equations (LAEs), ordinary differential equations (ODEs), and partial differential equations (PDEs) are more tractable than those obtained through earlier methods of underactuated mechanical system controller design[16,17].

The objective of this work is to apply the DLA method to control the unmanned hovercraft system. The Lyapunov based stability is designed and simulated to illustrate the efficacy of the designed control law.

2 Dynamic and modeling analysis of the system

The dynamic system was defined so that there are n generalized coordinates and m actuators, and the equations of motion governing the behavior of the autonomous hovercraft with nonholonomic constraints are determined from the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L(\mathbf{q},\dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}}\right)-\frac{\partial L(\mathbf{q},\dot{\mathbf{q}})}{\partial \mathbf{q}}=[\mathbf{M}(\mathbf{q})]\ddot{\mathbf{q}}+[\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})]\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q})=\boldsymbol{\tau} \tag{1}$$

where, $\mathbf{q}\in\Re^n$ represents the vector of generalized coordinates, with x, y and ψ representing the generalized hovercraft position and orientation in the earth fixed coordinates, respectively. $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}\in\Re^n$ represent velocities, and accelerations, respectively, for the n=3 degrees of freedom of the hovercraft system. $L(\mathbf{q},\dot{\mathbf{q}}):\Re^{2n}\rightarrow\Re$ is the Lagrangian defined as the kinetic energy minus the potential energy of the system. The right-hand side of Equation (1), specified as $\boldsymbol{\tau}\in\Re^n$, consists of the actuation for the degrees of freedom. It is assumed that the degrees of freedom are ordered so that the first m elements of the right side vector contain the nonzero inputs. For an underactuated system, only m of the inputs are nonzero where $m<n$. In the dynamic equations of motion (1), $[\mathbf{M}(\mathbf{q})]\in\Re^{n\times n}$ is the positive definite mass and/or inertia matrix, $[\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})]\dot{\mathbf{q}}\in\Re^n$ consists of centripetal and Coriolis forces and/or moments, and $\mathbf{G}(\mathbf{q})\in\Re^n$ consists of forces and/or moments stemming from gradients of conservative fields.

The requirement of the control law is to stabilize the system and in order to achieve this, Lyapunov's second method is applied for its development. The control challenge arises from the nonlinear nature of the governing equations and the underactuation. The candidate Lyapunov function is made of intrinsically positive quantities, part of which is described as a quadratic matrix product. The goal of this effort is to use a trial Lyapunov function

$$\mathrm{V}(\mathbf{q},\dot{\mathbf{q}})=\frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}[\mathbf{K_D}]\,\dot{\mathbf{q}}+\Phi(\mathbf{q}) \tag{2}$$

where, $\mathrm{V}(\mathbf{q},\dot{\mathbf{q}}):\Re^{2n}\rightarrow\Re$ is the candidate Lyapunov function; $\Phi(\mathbf{q})$ is a real scalar potential function of the generalized coordinates; $K_D\in\Re^{n\times n}$ is a symmetric, positive matrix defined as the product

$$\mathbf{K_D}=\mathbf{P}(\mathbf{q})\,\mathbf{M}(\mathbf{q}) \tag{3}$$

and where $\mathbf{P}(\mathbf{q})\in\Re^{n\times n}$ is a matrix defined so that $\mathbf{K_D}$ has the previously mentioned specified properties.

The time derivative of the candidate function is made non-positive and this concept is the basis for the Lyapunov application to nonlinear control problems. The time derivative of Equation (2), together with the equations of motion results in an equation that is solved by a matching method. When this method is applied, the terms quadratic in the velocities are grouped together obtaining a set of linear ordinary differential equations (ODEs). These equations are called the first matching condition[13-15].

Grouping terms which are linear in the velocities results in linear algebraic equations (LAEs) and these equations are called the second matching condition.

The third matching condition involves only position coordinates resulting in linear partial differential equations (PDEs).

3 Hovercraft model

Figure 3 shows the geometry of the hovercraft.

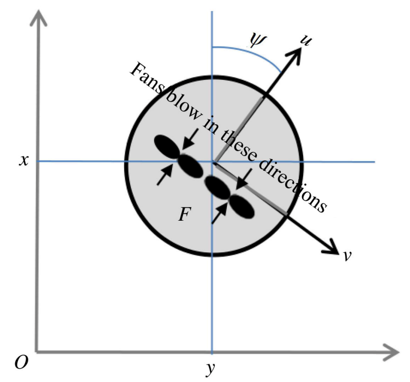


Figure 3 The simplified hovercraft model

From the Figure 3, the position vector is defined in the earth fixed coordinates as $\mathbf{q}=[x,y]^{\mathrm{T}}\in\Re^2$, $\psi\in\Re$ represents the hovercraft orientation in the earth fixed coordinates, $u\in\Re$ and $v\in\Re$ are linear velocities in surge and sway directions, respectively, $r\in\Re$ is the angular velocity, and $\mathbf{F}\in\Re$ is the control force in surge.

The kinematics in the inertial system that involves the hovercraft can be expressed as

$$\left.\begin{aligned}\dot{x} &= \cos(\psi)u - \sin(\psi)v\\ \dot{y} &= \sin(\psi)u + \cos(\psi)v\\ \dot{\psi} &= r\end{aligned}\right\} \tag{4}$$

Manipulating and rearranging terms from Equation (4), expressions for the local velocities are found[18-20].

$$\left.\begin{aligned}v &= \dot{y}\cos(\psi) - \dot{x}\sin(\psi)\\ u &= \dot{x}\cos(\psi) + \dot{y}\sin(\psi)\\ \dot{\psi} &= r\end{aligned}\right\} \tag{5}$$

Using Equation (5), the kinetic and potential energy are derived to define the Lagrangian L=T−V and by applying the Euler-Lagrange formulation, Equation (1) is redefined as

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{q} = \begin{bmatrix}\mathbf{F}\\ \boldsymbol{\tau}\\ 0\end{bmatrix} \tag{6}$$

where, $\mathbf{F}\in\Re$, as mentioned above, denotes the control force in the surge direction and $\boldsymbol{\tau}\in\Re$ denotes the control torque in yaw. The control torque is a function of **F** and its perpendicular distance from the center of the fan to the center of mass of the hovercraft[21].

Note that in order to obtain a simple model capturing essential nonlinearities of the hovercraft, the inertia matrix was assumed to be diagonal and constant for simplicity. If **M** is constant, the Coriolis and centripetal matrix is equal to zero. The hydrodynamic damping was cancelled because it is not used in controlling the system.

To use a direct Lyapunov method for designing a control law, the time derivative of Equation (2) is computed and it produces

$$\begin{aligned}\dot{V} &= \dot{q}^T\mathbf{K}_D\ddot{q} + \frac{1}{2}\dot{q}^T\dot{\mathbf{K}}_D\dot{q} + \dot{q}^T\nabla\varphi(\mathbf{q})\\ &= \dot{q}^T\mathbf{K}_D\mathbf{M}^{-1}(\mathbf{q})\left(-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s} - \begin{bmatrix}\mathbf{F}\\ \boldsymbol{\tau}\\ 0\end{bmatrix}\right)\\ &+\frac{1}{2}\dot{q}^T\dot{\mathbf{K}}_D\dot{q} + \dot{q}^T\nabla\varphi(\mathbf{q}) = -\dot{q}^T\mathbf{K}_v\dot{q}\end{aligned} \tag{7}$$

Following the procedures of the literature [22], Equation (7) is decomposed into three matching equations. Since the $\mathbf{K}_D$ is a constant matrix, this leads to the first marching condition as

$$\dot{\mathbf{K}}_D = 0 \tag{8}$$

The second matching equation, after expressing **F** as $F = F_1\dot{q}$ and rearranging the $\dot{q}$ terms, is

$$\begin{bmatrix}\mathbf{F}_1\\ \boldsymbol{\tau}_1\\ 0\end{bmatrix} = -\mathbf{P}(\mathbf{q})^{-1}\mathbf{K}_v \tag{9}$$

for which the solution is

$$\mathbf{K}_v = \sum_{i=1}^{m}\alpha_i\mathbf{P}_i\mathbf{P}_i^T \tag{10}$$

where the α_i are constants chosen so that $\mathbf{K}_v$ is positive semi-definite and $\mathbf{P}_i$ is the i^{th} column of $\mathbf{P}(\mathbf{q})$. The control law contribution from the second matching condition is the product of $\mathbf{F}_1$ and $\dot{q}$.

The third matching equation, where the first m equations in Equation (6) are used to determine the control law contribution —while the last $n–m$ rows of the equation provide linear, first order partial differential equations for the potential as seen in

$$-\mathbf{P}(\mathbf{q})\mathbf{G}(\mathbf{q})+\mathbf{P}(\mathbf{q})\begin{bmatrix}\mathbf{F}_2\\ \boldsymbol{\tau}_2\\ 0\end{bmatrix}+\nabla\Phi(\mathbf{q})=0 \qquad (11)$$

where, $\mathbf{G}$=0.

In taking the time derivative of the candidate Lyapunov function, the potential, defined in Equation (2) is assumed to be a function of the generalized positions $\mathbf{q}$ alone. At this time it is important to mention that the potential and the Hessian of the potential are needed in order to assure the stability condition of the system. The Hessian which denotes the second derivative of the potential with respect to the generalized coordinates, must be positive definite, and it is given by

$$\mathbf{H}=\begin{bmatrix}\dfrac{\partial^2\Phi(\mathbf{q}_1,\mathbf{q}_2)}{\partial\mathbf{q}_1\partial\mathbf{q}_1} & \dfrac{\partial^2\Phi(\mathbf{q}_1,\mathbf{q}_2)}{\partial\mathbf{q}_1\partial\mathbf{q}_2}\\ \dfrac{\partial^2\Phi(\mathbf{q}_1,\mathbf{q}_2)}{\partial\mathbf{q}_2\partial\mathbf{q}_1} & \dfrac{\partial^2\Phi(\mathbf{q}_1,\mathbf{q}_2)}{\partial\mathbf{q}_2\partial\mathbf{q}_2}\end{bmatrix} \qquad (12)$$

and the necessary condition on $|\mathbf{H}|$ is

$$\det(\mathbf{H})>0 \qquad (13)$$

for which all of the eigenvalues are positive. The method to solve the third matching equation is similar to the matching equations developed for stabilization as shown in literature [22].

The different parameters are chosen such that the eigenvalues of the linearized system are the same as those chosen for stabilization.

The Hessian of the potential is tested so that the potential is concave upward at the equilibrium point. It is a convenient way to choose the parameters.

The stabilization will be achieved, once all the stated constrains are satisfied. The candidate Lyapunov function also needs to be tested in order to verify that the Lyapunov function is positive definite and its first derivative with respect to the time is non-positive. Testing the control law through simulation will verify the reliability of the process. To do the numerical simulation, the derivation of the quantities $\mathbf{K_D}$, $\mathbf{K_v}$ the potential, the control inputs, and the coefficients are brought to Matlab from MAPLE where the symbolic manipulation was performed.

3 Results and discussion

In order to evaluate the results of the different proposed control laws, some simulations were performed using the following numerical values. The corresponding element of the $\mathbf{M}$ matrix for the hovercraft system are $m_1=m_2$=2.1 kg, J=0.0287 kg·m^2. The constant α=1e-10, was chosen such that the $\mathbf{K_v}$ matrix is positive semi-definite. The corresponding element of the $\mathbf{K_D}$ matrix from the proposed control are K_{D11}=26.496, K_{D21}= 0.01, K_{D31}=800, K_{D22}=10 K_{D33}=90448, K_{D32}=K_{D31}.

The simulation was performed in Matlab/Simulink with the corresponding structure illustrated in Figure 4.

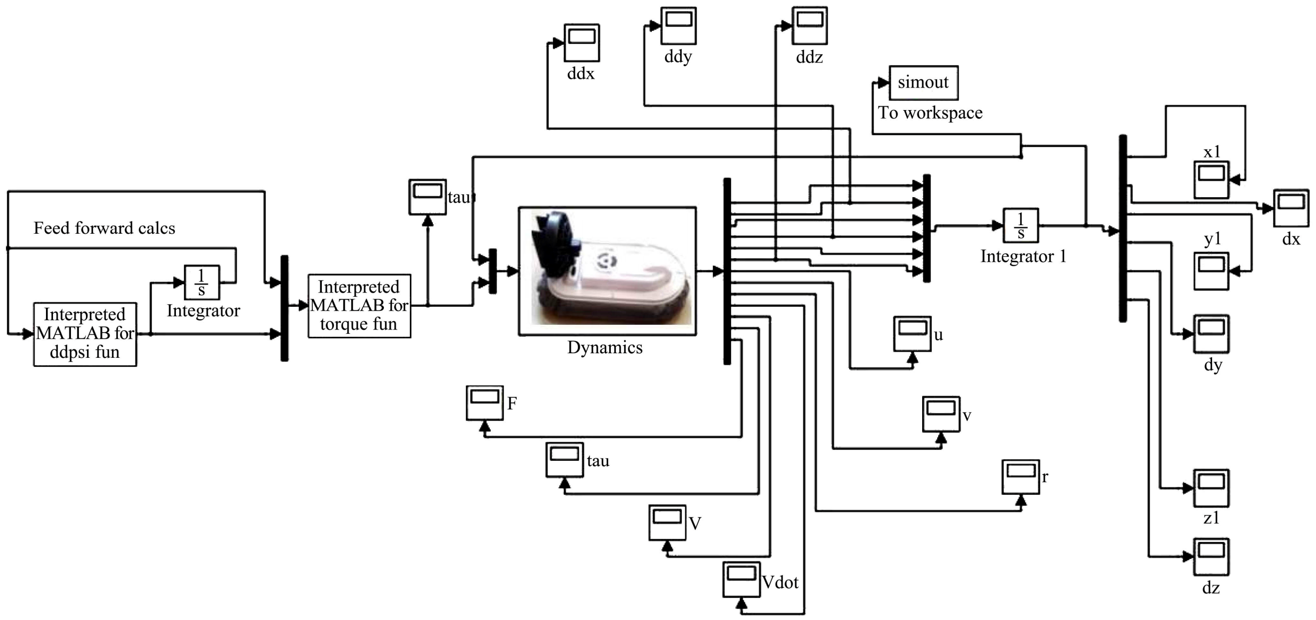
The control law designed is applied to the hovercraft to drive the states from a given initial condition to the origin and stabilizing them at that point. Numerical simulation confirms that the nonlinear control law stabilizes the system. The simulation results presented in the plots of Figures 5 and 6 illustrate the hovercraft position and velocity as well as the orientation angle and angular velocity as a function of time, respectively.

Figures 7 and 8 show the upward concave shape of the potential. It demostres the stability of the system, thus the eigenvalues of the Hessian are positive. From the plot in Figure 7, it is seen that y is contained in the interval (-30, 30), while the x position is contained in the interval (-1, 1) where $\Phi(x, y)$ is a convex function. Values of x and y outside the stated region also produce stable behavior.

Figure 8 presents a 3D plot of the potential for the interval (-30, 30) for ψ (psi) and the interval (-26, 26) for y, such that $\Phi(y, \psi)$ is a convex function.

Figures 9 and 10 show the Lyapunov function performance and its first time derivative, as well as the control law. The behavior shown in Figures 9 and 10 demonstrate the validity of the Lyapunov candidate function because it is monotonically decreasing with time as expected for the stability condition.

The behavior of the control law to stabilize the hovercraft system is shown in Figure 11 and Figure 12 for F and for τ, respectively.



Note: u, v and r converge to zero.

Figure 4 Control architecture

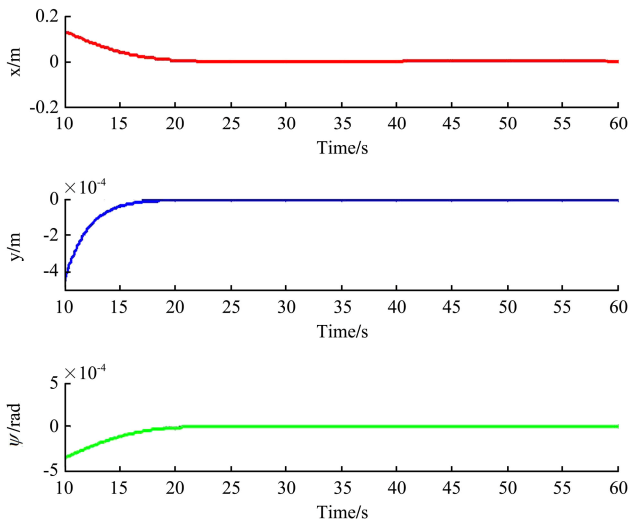


Figure 5 Stabilization of the Hovercraft (Generalized position *x, y* and orientation *ψ*)

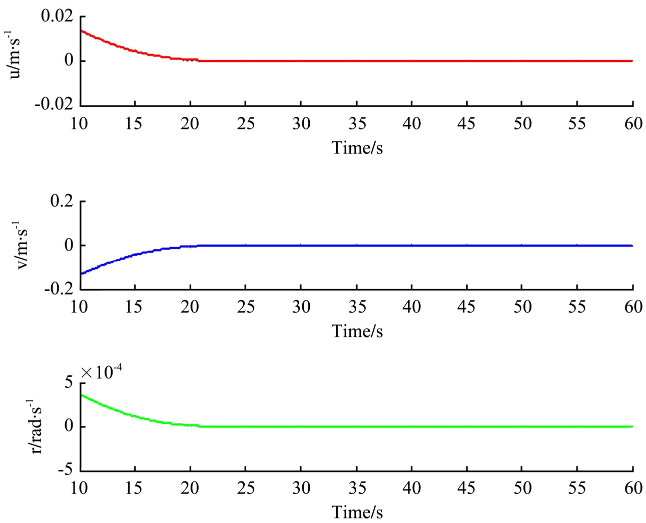


Figure 6 Velocity variables (*u, v* and *r*) for stabilization of the hovercraft

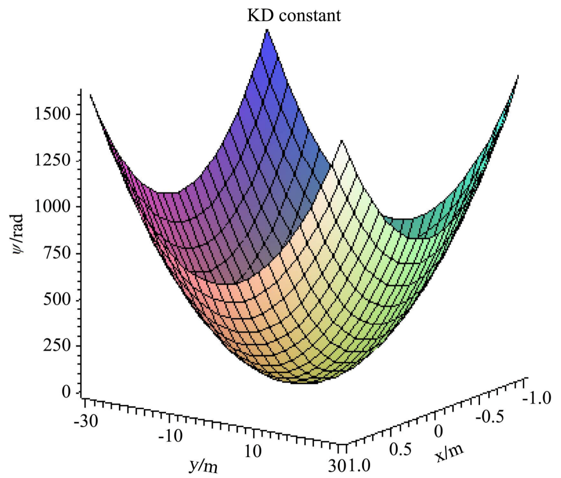


Figure 7 Hovercraft Potential (*x*-*y* plane)

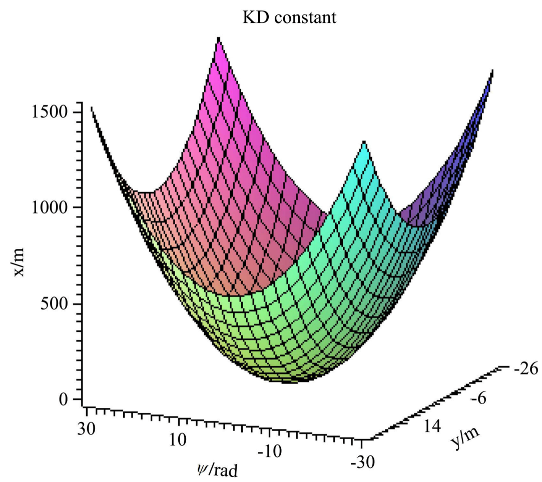


Figure 8 Hovercraft potential (*y*-*ψ* plane)

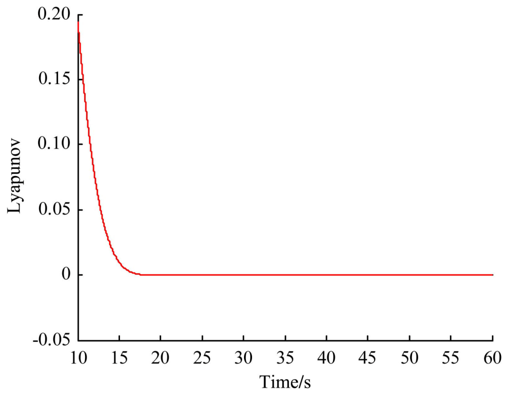


Figure 9 Lyapunov time history

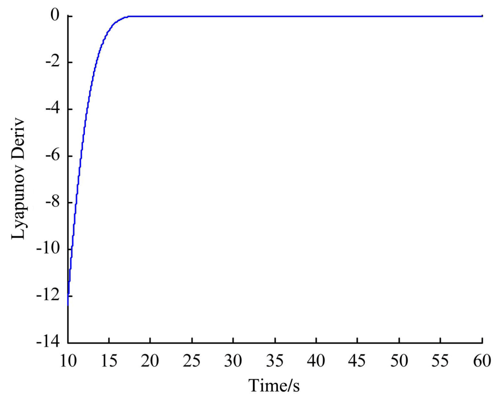


Figure 10 Lyapunov time derivative

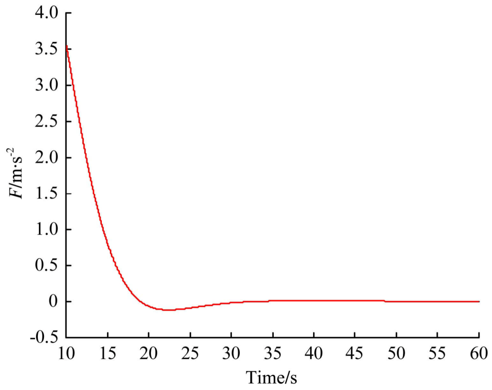


Figure 11 Control law (F)

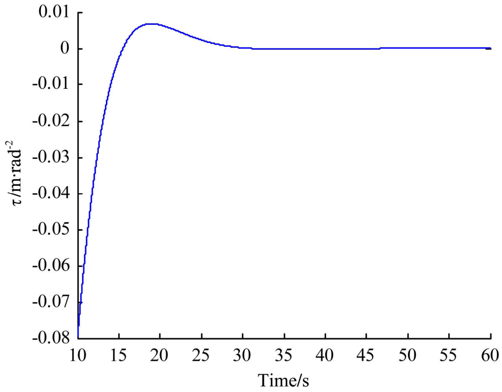


Figure 12 Control law (τ)

4 Conclusions

Agricultural applications such as weed control, crop preparation, and more efficient use of fertilizers are procedures that involve low environmental and operator exposure risk together with the requirement of successful crop production. The operations of fertilizer, pesticide, and herbicide applications must be made easier and safer by using autonomous vehicles for the performance of these tasks. When using an autonomous hovercraft vehicle, the dispersion of the agricultural quantities will be highly effective compared to aerial vehicles due to the surface proximity on which they act and the dispersion can be accomplished without the risk of operator exposure. In order to successfully achieve this objective, a nonlinear controller based on direct Lyapunov approach was designed for an unmanned hovercraft.

This work introduces the methods which are applied to a model that can be used to simulate the behavior of an underactuated system with three degrees of freedom and two control inputs. A scheme based on a Lyapunov approach to stabilize the surge, sway and angular velocity of yaw has been proposed as a means to design the controller. The model and the controller are tested in the Matlab/Simulink environment to demonstrate the hovercraft stabilization.

The simulated model was used to test the control law showing the stability performance. The designed control produced good performance results showing a fast response. It must be acknowledged that the hovercraft dynamics will change with the quantity of fuel and agricultural products stored on board where the lighter the hovercraft becomes, the quicker the positioning transients. One advantage in this work was the symbolic manipulator which allowed the modifications of the parameters and quick transferal of the symbolic manipulator output to Matlab/Simulink environment to produce new simulation results.

Modeling and control technologies are required to assure that the prototype will perform safely, reliably, and robustly in the presence of disturbances and changing weight. The designed control law has been implemented on a hovercraft prototype equipped with an Arduino microcontroller together with a GPS sensor for position measurement and an inertial measurement unit (IMU) for orientation measurement. The goal is to analyze the behavior of the propulsion drivers and to determine whether it is sufficient to direct and stabilize the hovercraft. This is the next step of this work which is currently in progress.

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