Development and experimental verification of a mathematical expression for the discharge rate of a semi-circular open channel

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Abstract: Semi-circular open channel plays an important role in various applications and the measurement of its discharge is of interests. In this study, theoretical formulae for free overflow in a semi-circular channel are developed and presented for the discharge and wetted area relationship. The traditional discharge formulation and available experimental data are used to verify and validate the proposed relationships. The discharges calculated by using the proposed relationship show very good agreement with the experimental data sets. The results from this study supply the basis for circular weir development.

Keywords: verification, discharge, open channel flow, weir, circular, wetted area, measurement

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1 Introduction

The measurement of stream discharges or flow rates in open channels with weirs of various designs has been a classic topic of interests to many practical engineers[1]. Formulae for the steady-state discharge as functions of hydraulic heads are presented in many standard civil[2] and chemical engineering[3] text books for weirs of simple geometric shapes, such as rectangular and triangular (or V-notch).

Kadlubowski et al.[1] stated many advantages of the circular weirs, such as free turning of the weir crest enabling it to be standardized by precise positioning and the ease of manufacture. Moreover, the weir crest does not have to be leveled in terms of cross section during installation, and the point of zero flow is readily determined. The use of semi-circular channels is common in sewer systems and tunnels. Since the hydraulic conditions in a pipe partly filled with water under atmospheric condition are the same as those in a semi-circular channel, research on the formulation of semi-circular weir flow is also of importance to that situation, particularly for environmental applications.

Dey[4-6] and Ahmad[7] conducted theoretical and experimental studies on free overflow in an inverted

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semi-circular channel. In fact, despite their advantages, circular weirs are not commonly used for measuring discharge, perhaps because of the cumbersome mathematics associated with them[1]. Specifically, the functional relationship between the hydraulic head and the volumetric liquid discharge incorporates elliptic integrals as described in the classic paper by Stevens[8] or is expressed with several variables including trigonometric functions. Consequently, in most cases it has not been considered to be convenient for practical engineering applications. The need for discharge measurements naturally raises the demand for a simple formula relating discharge to the wetted area.

According to the energy equation for open channel flow[9], the liquid volumetric discharge across the weir is simply proportional to the 3/2 power of the height of the liquid crest for a rectangular weir, and to the 5/4 power for a triangular weir, as in Equation (1a) and Equation (1b), respectively:

$$Q_R \propto A^{\frac{3}{2}} \tag{1a}$$

$$Q_T \propto A^{\frac{5}{4}} \tag{1b}$$

Where: Q_R/Q_T is the volumetric discharge for rectangular/triangular weir, m^3/s; A the wetted area, m^2.

Figure 1 shows the geometrical relationships of the cross-section of the three different weir channels, i.e. rectangular, triangular and semi-circular channels. Figure 1 appears to state that there should exist a similar Q-A relationship for semi-circular weir or a partly filled circular weir, but with a different exponential value of between 5/4 (for a triangular weir) and 3/2 (for a rectangular weir). Depending upon the shape of the open channel, the appropriate equation can be adopted to determine the discharge rate of the steady-state inflow from an upstream source, given the wetted area of the liquid therein.

A mechanic-electronic sensor for automatic measurement of sediment-laden discharge from erosion-runoff plots was presented by Qu et al.,[10], in which a load cell is used to sense the mass of water in a measuring pipe section. For the given mechanical structures, the sensed mass of water in the measuring pipe section is proportional to the discharge/flow rate. The hydraulic principles and computational model of the sensor were based on a function relation between discharge and the mass or the cross-sectional area of water flow.

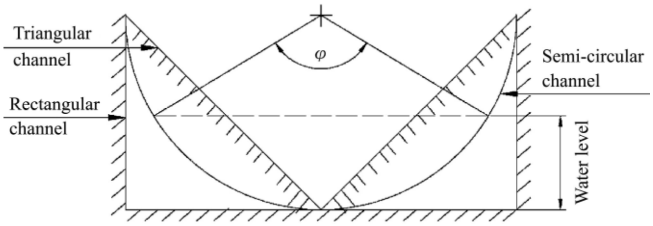

Figure 1 Cross-sectional diagram for triangular, rectangular and semi-circular channels

The objectives of this paper are: 1) To develop the theoretical relationship between the discharge and the wetted area of a semi-circular open channel based on hydraulic characteristics with appropriate mathematical approximations; 2) To validate the newly-developed equations and the parameters with the data from various sources.

2 Mathematical model derivation

The relationship among the discharge rate, wetted area and hydraulic radius are given by the Chezy and Manning equations (Equations (2) and (3)) as follows:

$$\frac{Q}{\sqrt{i}} = AC\sqrt{R} \tag{2}$$

$$C = \frac{1}{n}R^{\frac{1}{6}} \tag{3}$$

Combining Equations (2) and (3) gives:

$$Q = \frac{1}{n}Ai^{\frac{1}{2}}R^{\frac{2}{3}} \tag{4}$$

Where: Q is the discharge rate of water, m^3/s; i is the hydraulic gradient, m/m; A is the cross-sectional area of water flow, m^2; C is the Chezy coefficient, m$^{1/2}$/s; R is the hydraulic radius, m; and n is the Manning coefficient, dimensionless.

Combining Equation (4) with Equation (1a) or Equation (1b) yields the R-A relationship for a rectangular weir (5a) and for a triangular weir (5b), respectively:

$$R = \sigma_1 A^{\frac{3}{4}} \tag{5a}$$

$$R = \sigma_2 A^{\frac{3}{8}} \tag{5b}$$

Where: σ_1 and σ_2 are proportional constants.

In a semi-circular channel or a pipe partly filled with water under no pressure, the hydraulic radius and wetted area are calculated by Equation (6) and Equation (7), respectively as follows:

$$R=\frac{d}{4}\left(1-\frac{\sin\varphi}{\varphi}\right) \tag{6}$$

$$A=\frac{d^2}{8}(\phi-\sin\phi) \tag{7}$$

Where: d is the diameter of the circular channel, m; and ϕ is the central angle of the wetted perimeter, radians (Figure 1). According to the relationship between the hydraulic radius and the wetted area of a rectangular weir and/or a triangular weir, the relationship for a semicircular channel or a pipe partly filled with water under no pressure is given as follows:

$$R=\sigma A^{\chi} \tag{8}$$

Where: σ is a constant; χ is a power index to be estimated further. Combining Equation (6) and Equation (7) with Equation (8) gives the following relationship:

$$\frac{R}{A^{\chi}}=\frac{\frac{d}{4}\frac{(\varphi-\sin\varphi)}{\varphi}}{\left(\frac{d^2}{8}\right)^{\chi}(\varphi-\sin\varphi)^{\chi}}=const \tag{9a}$$

Rearranging the equation gives:

$$\frac{R}{A^{\chi}}=\frac{2^{3\chi-2}(\varphi-\sin\varphi)^{1-\chi}}{d^{2\chi-1}\varphi}=const \tag{9b}$$

Simplifying Equation (9b) gives:

$$\frac{R}{A^{\chi}}=K_1\frac{(\varphi-\sin\varphi)^{1-\chi}}{\varphi}=const \tag{9c}$$

Where: K_1 is a proportional constant.

The Taylor series expansion for $\sin\varphi$ is written at the point $\varphi=0$ as follows:

$$\sin\varphi=\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\ldots\ldots+(-1)^n\frac{\varphi^{2n-1}}{(2n-1)!} \tag{10}$$

Arbitrary truncation of the series gives an error, which is quantified by the residues of the truncated series, expressed as follows:

$$R_n(\phi)=\frac{f^{(n+1)}(\xi)}{(n+1)!}(\phi)^{n+1}\le\frac{|\phi|^{n+1}}{(n+1)!}\le\frac{|\pi|^{n+1}}{(n+1)!}$$
$$\xi\in(0,\phi) \tag{11}$$
$$\phi\in(0,\pi)$$

With different truncated terms, the maximum residue in the range between 0 and π is estimated as shown in Table 1.

Table 1 Residual term and the estimate of the maximum R_n from Equation (11)

N	$\sin\varphi$	Max($R_n(\varphi)$)
1	φ	$\frac{\pi^2}{2!}=4.935$
3	$\varphi-\frac{\varphi^3}{3!}$	$\frac{\pi^4}{4!}=4.059$
5	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}$	$\frac{\pi^6}{6!}=1.335$
7	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}$	$\frac{\pi^8}{8!}=2.353\times10^{-1}$
9	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\frac{\varphi^9}{9!}$	$\frac{\pi^{10}}{10!}=2.581\times10^{-2}$
11	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\frac{\varphi^9}{9!}-\frac{\varphi^{11}}{11!}$	$\frac{\pi^{12}}{12!}=1.930\times10^{-3}$
13	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\frac{\varphi^9}{9!}-\frac{\varphi^{11}}{11!}+\frac{\varphi^{13}}{13!}$	$\frac{\pi^{14}}{14!}=1.046\times10^{-4}$
15	$\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\frac{\varphi^9}{9!}-\frac{\varphi^{11}}{11!}+\frac{\varphi^{13}}{13!}-\frac{\varphi^{15}}{15!}$	$\frac{\pi^{16}}{16!}=4.303\times10^{-6}$

For an error less than 4.303E-6, $\sin\varphi$ is given as follows:

$$\sin\varphi=\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\frac{\varphi^9}{9!}-\frac{\varphi^{11}}{11!}+\frac{\varphi^{13}}{13!}-\frac{\varphi^{15}}{15!} \tag{12}$$

Substituting Equation (12) into Equation (9c) yields:

$$\frac{R}{A^{\chi}}=K_1\frac{\left(\frac{\varphi^3}{3!}-\frac{\varphi^5}{5!}+\frac{\varphi^7}{7!}-\frac{\varphi^9}{9!}+\frac{\varphi^{11}}{11!}-\frac{\varphi^{13}}{13!}+\frac{\varphi^{15}}{15!}\right)^{1-\chi}}{\varphi} \tag{13}$$

To simplify the formula, the numerator in Equation (13) is used to build another function (Equation (14)). It is used to regress the computed data from different φ values ranging from 0 to π, shown in Figure 2.

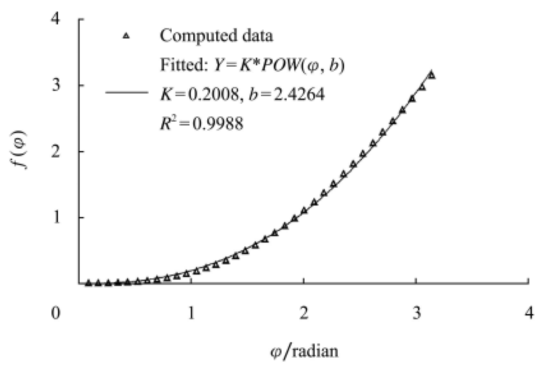

Figure 2 Curve of the polynomial regression for the sine function (Equation (14)) against the independent variable

With a very high coefficient of determination (0.9988), the regression coefficient in Equation (15) was determined to be K_2 = 0.2, the exponential index, b to be 2.43.

$$f(\varphi)=\frac{\varphi^3}{3!}-\frac{\varphi^5}{5!}+\frac{\varphi^7}{7!}-\frac{\varphi^9}{9!}+\frac{\varphi^{11}}{11!}-\frac{\varphi^{13}}{13!}+\frac{\varphi^{15}}{15!} \quad (14)$$

$$f(\varphi)=K_2\varphi^{2.43} \quad (15)$$

Substituting Equation (15) into Equation (13) yields:

$$\frac{R}{A^{\chi}}=K_1\frac{(K_2\varphi^{2.43})^{1-\chi}}{\varphi}=const \quad (16a)$$

$$\frac{R}{A^{\chi}}=K_3\varphi^{(1.43-2.43\chi)}=const \quad (16b)$$

Equation (16b) requires that the coefficient χ must be 0.5885 which gives the exponential index in Equation (16b) to be zero. The values of R/d and A/d^2 are normally provided as tabulated data sets[9]. The data sets fom experiments with the partly filled circular pipe, as given by Armando[9], were regressed with Equation (8) and shown in Figure 3. The two sets of data as given by Armando[9] and those computed with Equation (8) are very well correlated, with the determination coefficient, R^2, being 0.9993. The regressive coefficient, σ, being 0.4453, with χ being 0.5994, which is very close to 0.5885, the value estimated by the mathematical method. This indicates that Equation (8) and the estimated parameter χ are rational and reasonable. The other two fitted curves in Figure 3 are those computed by giving an exponential index value of 3/4 (for a triangular weir) and 3/8 (for a rectangular weir) in Equation (8). The curve regressed with the formula for circular weir (Equation (8)) lies between those for triangular weir and rectangular weir, which just agrees with the concept illustrated in Figure 1.

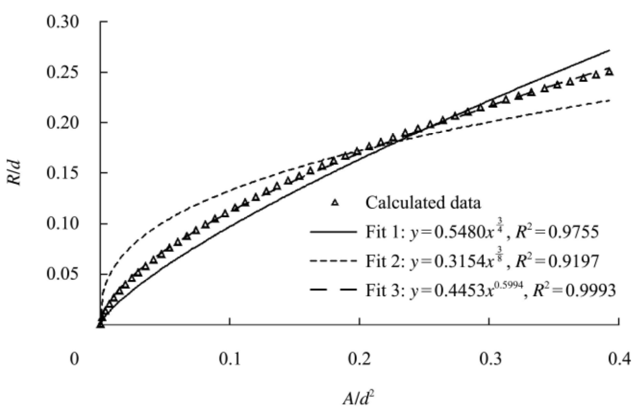

Figure 3 Regressed relationship between the hydraulic radius R/d and the wetted area A/d^2 of a partly filled circular pipe

The exponential index in Equation (8) is 0.5994 from regression and 0.5885 from the hydraulic and mathematical concepts. They do not differ significantly from 3/5, as far as engineering applications concerned. Therefore, the relationship between A and R for a semicircular weir or a partly filled circular pipe is given as follows:

$$R=\sigma A^{3/5} \quad (17)$$

Substituting Equation (17) into Equation (4) yields:

$$Q=\frac{1}{n}i^{\frac{1}{2}}\sigma^{\frac{2}{3}}A^{\frac{7}{5}} \quad (18a)$$

Because the hydraulic gradient i and the manning coefficient n and parameter σ are all constants, independent with the cross-sectional area of the water flow, A, they can be expressed as an integrated coefficient and Equation (18a) becomes:

$$Q=\alpha A^{\frac{7}{5}} \quad (18b)$$

The exponential index of 7/5 is indeed between 3/2 and 5/4, as predicted in the early sections, being between 3/2 for a rectangular weir and 5/4 for a triangular weir, respectively.

3 Validation and discussion

To validate the approach presented above, data from two sources, computed and measured, are used to check the derived equations.

3.1 Verification with the discharge function for semi-circular channel

Weirs are described as sharp-crest weirs or thin-plate weirs when the upper edge over which the flows, i.e. the crests, are very narrow. For a circular sharp-crested weir, the discharge is given by Armando[9] as follows:

$$Q=\mu\phi d^{\frac{5}{2}} \quad (19)$$

Where: d is the diameter, in dm and Q is the discharge rate, in dm^3; ϕ is a function of the water level; and μ is calculated from Equation (20), in which, h is the water head, the value of h/d is given in Table 2:

$$\mu=0.555+\frac{d}{110h}+0.041\frac{h}{d} \quad (20)$$

Limits of the equation are that the approach velocity must below $V^2/2g\approx0$; $a>r$, with a minimum of 0.10 m; $b>r$; $h/d\geqslant0.10$; $h\geqslant0.03$ m; the water downstream must be at least 0.05 m below the crest. Figure 4 illustrates these variables.

Table 2 Data sets of parameters used in Equations (19) and (20)

h/d	ϕ	h/d	ϕ
0.05	0.0272	0.3	0.9119
0.1	0.1072	0.4	1.5713
0.2	0.4173	0.5	2.3734

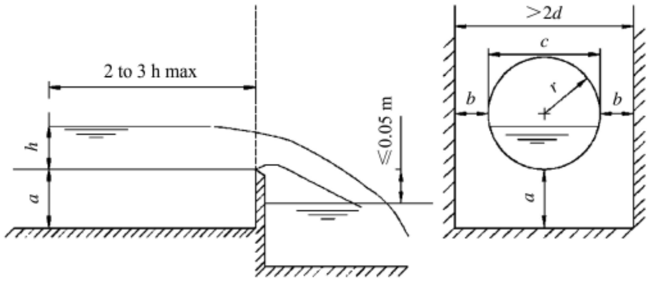

Figure 4 Diagram of a circular shape-crested weir

It is of great interest to verify the relationship between discharges calculated with the two different formulae, i.e., Equation (18b) and Equation (19). For simplification, a theoretical example for a circular weir assumes that the diameter of the circular transect, d, is 1 m and the discharge, Q_1, is then determined for various water levels by using Equation (19). Likewise, the discharge, Q_2, under a wetted area, A, is determined from Equation (18b) taking the theoretical value of 1 for the coefficient, α. The calculated values for Q_1 and Q_2 are shown in Table 3.

Table 3 Discharge rates computed with different models, i.e., Equations (18b) and (19)

h/d	ϕ /radians	d /dm	μ	Q_1 /$m^3 \cdot s^{-1}$	A /m^2	Q_2 /$m^3 \cdot s^{-1}$
0	0	1	0	0	0	0
0.025	0.0136	1	0.9197	0.0110	0.0052	0.0007
0.050	0.0272	1	0.7389	0.0128	0.0147	0.0028
0.075	0.0672	1	0.6793	0.0256	0.0268	0.0065
0.10	0.1072	1	0.6500	0.0365	0.0409	0.0117
0.15	0.2622	1	0.6218	0.0799	0.0739	0.0267
0.20	0.4173	1	0.6087	0.1206	0.1118	0.0476
0.25	0.6646	1	0.6016	0.1866	0.1535	0.0739
0.30	0.9119	1	0.5976	0.2518	0.1982	0.1054
0.35	1.2416	1	0.5953	0.3395	0.2450	0.1415
0.40	1.5713	1	0.5941	0.4275	0.2934	0.1818
0.45	1.9723	1	0.5937	0.5356	0.3428	0.2258
0.50	2.3734	1	0.5937	0.6445	0.3927	0.2727

The Q_1 and Q_2 values from Table 3 were compared in Figure 5. There was a very good, linear correlation between Q_1 and Q_2 values, with a determination coefficient, R^2, being 0.9992. This indicates that the derived formula for discharge, expressed as a function of the wetted area, A, and the exponential parameter, are both reasonable and practicable.

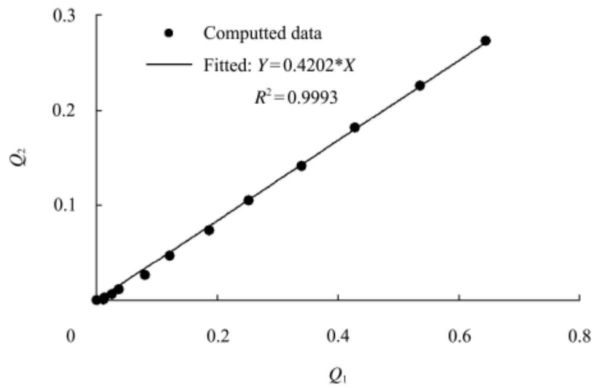

Figure 5 Comparison of flow rates computed by the traditional discharge formula and the proposed model

In practice, circular weirs have various values of α, dependent on their construction material and size, which need to be determined by calibration using Equation (18b). The value of α does not affect the fact that a linear relationship exists between the discharges calculated using the two Equations (18b) and (19); it just affects the value of the slope parameter, determined as 0.42 in our theoretical example (Figure 5).

3.2 Verification with measured data

The mechanic-electronic sensor proposed by Qu et al.[10] estimates discharge rates by means of sensing the weight of water in a measuring pipe. The force F, on the load cell, is determined by the water volume in the weighing section. The relationship among weight, volume of water in the measuring pipe and the force on the load cell from the water in the measuring pipe is determined by $F \propto W \propto V$. Based on the relationship between the discharge and the wetted area in Equation (18b), the relationship between the sensed weights of the water, F, and the discharge, Q, is given as follows:

$$Q = f(F) = \alpha_1 F^{\frac{7}{5}} \tag{21}$$

Where: α_1 is the proportional coefficient. Based on hydraulic theory and mathematical approximation, the exponential index should be equal to χ in Equation (8), 7/5. The derivation of Equation (21) is given in detail by Qu et al[10]. In order to validate Equation (18b), experiments were conducted to verify Equation (21), which was based on Equation (18b). If Equation (21) is reasonable, Equation (18b) is verified, albeit indirectly.

To calibrate the model, a flume, equipped with a load cell attached to a data logger, was used in which the inflow of tap water, and hence the discharge rate, is regulated by a manually operated valve. The discharge was measured for randomly selected rates ranging between 0 and 3,000 mL/s. A series of discharges were measured by collecting the water flow in containers, over timed intervals. For each valve setting used to control the water inflow rate, five runoff samples were taken to determine the mean discharge. The data logger recorded the load cell's outputs for each set of discharges.

Outputs, F from the load cell were used following Equation (21) to calculate discharges. In order to compare with previous study, other two exponential index, χ, values, i.e., 3/2 for rectangular and 5/4 for triangular channel were also used instead of 7/5 in Equation (21) to compute discharge, respectively. The measured discharge values of the clear water were then regressed against these calculated values (Figure 6). The determination coefficients for the three different χ values (3/2, 7/5, and 5/4) were 0.9802, 0.9851 and 0.9830, respectively.

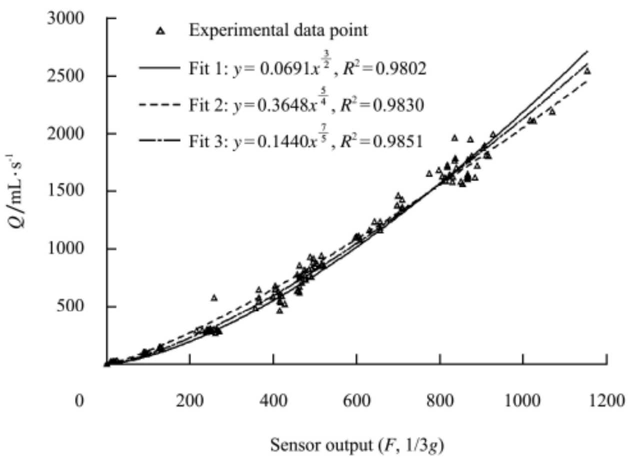

Figure 6 Experimental data and fitted curves, calculated using three different exponential indices

The regressed results (Figure 6) indicated that the sensor's output, F was very well correlated with the measured discharge (R^2 = 0.9851), which indicates that Equation (21) is rational. Further, it is proven that Equation (18b) should theoretically be a good function by which to quantify the relationship between discharge and the wetted area.

Laboratory experiments were also used to verify the relationships established above. The constant rainfall event used an intensity of 75 mm/h and lasted for ten minutes. The variable rainfall events involved the gradual, step-by-step increases in rainfall intensities from 30 to 70 mm/h in increments of 10 mm/h and three minutes periods. The data collected from that study were manipulated with different exponential index.

The laboratory model of the watershed was shown in Figure 7. The runoff discharges were measured with the flumes and were also manually-measured

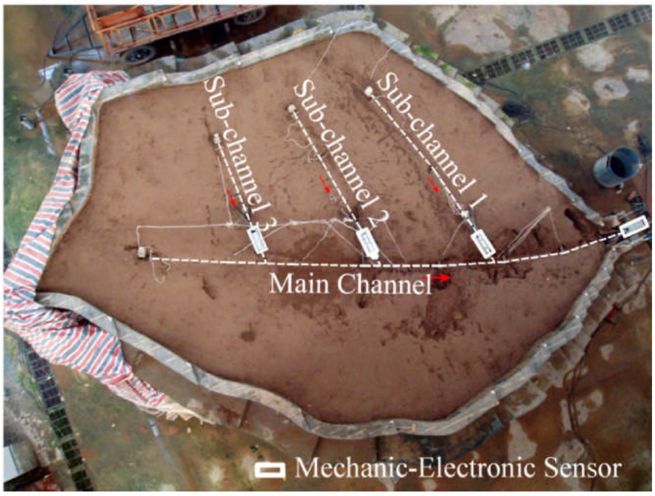

Figure 7 Aerial view of the laboratory watershed

The double-mass curve method was used to check the differences between the manually measured and the flume-measured data sets. The double-mass curves are shown in Figures (8a) and (8b) for the rainfall events. The two groups of data agreed well for both rainfall events, with a regressive coefficient of 0.9939 and a determination coefficient of 0.9997 for the rainfall event of constant intensity and a regressive coefficient of 1.0482 and a determination coefficient of 0.9998 for the rainfall event of variable intensities. The data sets indicated that there was no significant difference between the manually measured and computed sensor data sets.

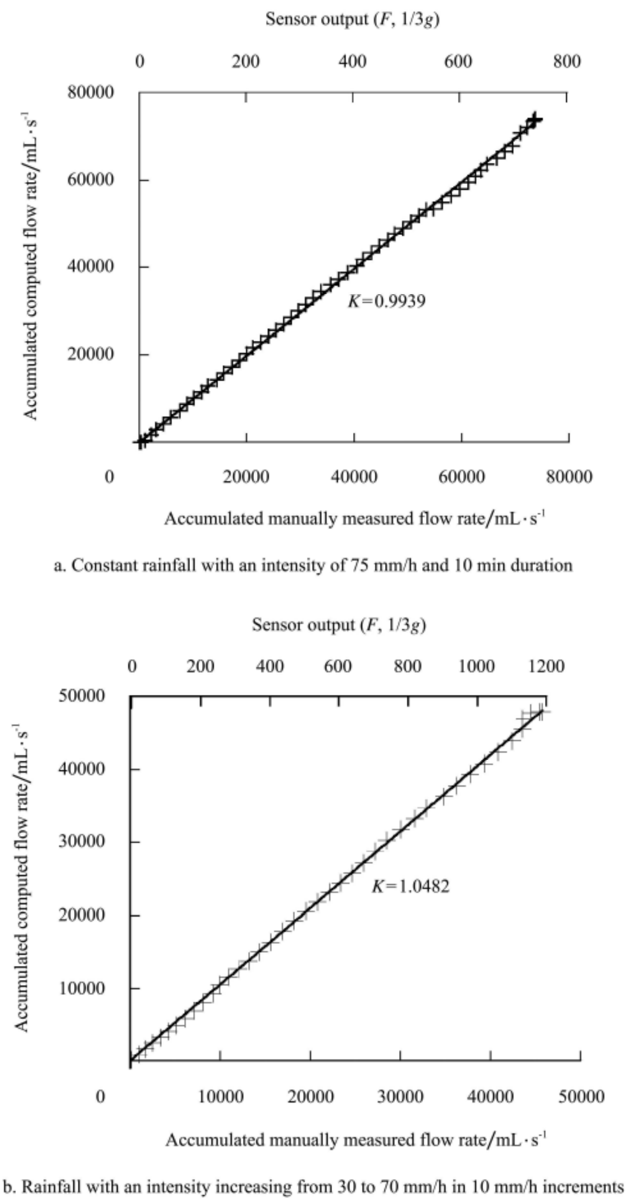

Figure 8 Double-mass curves of accumulated sensor-data, computed flow rates vs. manually measured flow rates

4 Discussion

From the theoretical point of view, the complex cross-sectional geometry complicates the analysis of circular weir. In the present approach, the problem is dramatically simplified by using a novel formulation and it makes the calibration much more convenient. Wahl et al.[11] and Uyumaz et al.[12] studied the flow in circular channels. The formulas obtained from their study were complex to determine the parameters because of the cross sectional geometry. In this study, the main focus is on the flow in a semi-circular shaped weir, which is also applicable to the situation when the flow is less than half full in a circular weir.

The model for semi-circular channels in this paper was valid when the weir length is not very long so that it is reasonable to use the averaged depth of the water flow in the flume. For practical application, calibration work is needed for a specific design of the weir. The precise of the measurement depends on the size of the design and in situ conditions, such as roughness, materials and temperature. This model should be helpful to the development of automatic measurement device.

In the previous studies, different χ *values* in Equation (18b) were suggested for triangular weir. Compared with the result by Qu et al.[10], the exponential index, χ, in Equation (8b) differs little from 3/2 to 7/5. This change makes improved measurements. Though 3/2 was a very close estimation and the improvement of regression is not very dramatic, the newly suggested index bears strong hydraulic and mathematical meanings.

5 Conclusions

A new method for the determination of discharge from the wetted area in semi-circular channels or circular pipes partly filled with water under no pressure was outlined. Theoretical and mathematical approximation procedures were applied to determine the formula parameters. The calculated discharges, using the proposed relationship, showed good agreement with data sets derived from the traditional formula and experiments.

These results suggest that the mathematical models and the procedures used for the determination of water discharge were rational. The model can also be applied to the problem of calculating the inflow of a fluid through an open channel or short pipe.

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Notation

Notation	Explanations	Units
Q	discharge rate	$m^3 \cdot s^{-1}$
A	cross-sectional area	m^2
I	hydraulic gradient	$m \cdot m^{-1}$
C	Chezy coefficient	$m^{1/2} \cdot s^{-1}$
R	hydraulic radius	m
n	Manning coefficient	dimensionless
σ_1	regression proportional coefficient of Eq.(5a)	$m^{-1/2}$
σ_2	regression proportional coefficient of Eq.(5b)	$m^{1/4}$
σ	regression proportional coefficient of Eq.(17)	$m^{-1/5}$
d	diameter of circular channel	m
ϕ	central angle of the wetted area	radian
x	regression proportional coefficient of Eq.(8)	dimensionless
K	regression proportional coefficient of Eq.(15)	dimensionless
μ	parameter of Eq.(19)	dimensionless
Φ	parameter of Eq.(19)	$m^{1/2}s^{-1}$
h	water level	m
α_1	proportional coefficient	$m^3 \cdot kg^{7/5} \cdot s^{-1}$
F	output of load-cell	kg

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